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2.26

Moderator: **helmut** [Moderator Control Panel]

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Author	Message

anthony.d.dennis23

offline

Member

Joined: Sat, 28 Mar 2020 15:16
Posts: 14

Post subject: 2.26 **Posted:** Tue, 31 Mar 2020 11:10

Because a sequence is included in all the other subsequences. Let a_{n_k} be a subsequence of the sequence a_n where $n_k = n, \forall n, k \in N$ therefore $a_{n_k} = a_n$ and because it is given that all subsequences converge to a, the sequence must also converge to a.

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helmut

online

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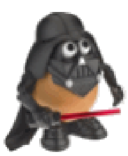

Joined: Sat, 26 Apr 2003 15:14
Posts: 2232
Location: El Paso TX (USA)

Post subject: Re: 2.26 **Posted:** Tue, 31 Mar 2020 13:27

The hypotheses state only that the **converging** subsequences converge to the same limit.

A proof will have to use the fact that the original sequence is bounded. Otherwise the sequence $a_n = n$ provides a counterexample. Why?

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

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solivas11

offline

Math Cadet

Joined: Mon, 30 Mar 2020 21:12
Posts: 7

Post subject: Re: 2.26 **Posted:** Tue, 31 Mar 2020 19:15

My attempt at 2.26. Feedback is appreciated...

a_n is a bounded sequence.
Suppose ALL converging subsequences: $a_{n_k} \longrightarrow b \in \mathbb{R}$, where $n, k \in \mathbb{N}$
Show $a_n \longrightarrow b$

Idea: Produce smaller and smaller bounds for a_n so, like a cobra, constrict it to b.

Via 2.24
$a_n \in [a, c]$ where $a < c$
$\exists a_{n_k} \longrightarrow b \in [a, c] : \forall \varepsilon > 0 \exists N \in \mathbb{N}$ such that $|a_{n_k} - b| < \varepsilon \forall n \geq N$
Since a_{n_k} is bounded, both $a_n, b \in [a, c] \forall n \geq N$

Let $\varepsilon_1 = |c-a|/2$
Let $a_1, c_1 \in [a,c]$ such that $|c_1 - a_1| = \varepsilon_1$, where $a \leq a_1 < b < c_1 \leq c$
For all converging subsequences, $\forall \varepsilon_1 > 0 \exists N_1 \in \mathbb{N}$ such that $\forall n \geq N_1 : |a_{n_k} - b| < \varepsilon_1$
Then $a_n \in [a_1, c_1] \forall n \geq N_1$

Let $\varepsilon_2 = |c-a|/(2^2)$
Let $a_2, c_2 \in [a_1, c_1]$ such that $|c_2 - a_2| = \varepsilon_2$, where $a_1 \leq a_2 < b < c_2 \leq c_1$
For all converging subsequences, $\forall \varepsilon_2 > 0 \exists N_2 \in \mathbb{N}$ such that $\forall n \geq N_2 : |a_{n_k} - b| < \varepsilon_2$
Then $a_n \in [a_2, c_2] \forall n \geq N_2$

.
.
.

Let $\varepsilon_m = |c-a|/(2^m)$
Let $a_m, c_m \in [a_{m-1}, c_{m-1}]$ such that $|c_m - a_m| = \varepsilon_m$, where $a_{m-1} \leq a_m < b < c_m \leq c_{m-1}$
For all converging subsequences, $\forall \varepsilon_m > 0 \exists N_m \in \mathbb{N}$ such that $\forall n \geq N_m : |a_{n_k} - b| < \varepsilon_m$
Then $a_n \in [a_m, c_m] \forall n \geq N_m$

By repeating the procedure above, where $a_n \in [a_m, c_m]$ of length $\varepsilon_m \forall n \geq N_m$ where $\varepsilon_m < \varepsilon_{m-1}$, then $a_n \longrightarrow b$

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anthony.d.dennis23
offline
Member
Joined: Sat, 28 Mar 2020 15:16
Posts: 14

Post subject: Re: 2.26 **Posted:** Thu, 02 Apr 2020 12:17

I don't see how $a_n \in [a_m, c_m]$ remains true. Are you saying that because $a_{n_k} \in [a_m, c_m]$ occurs that $a_n \in [a_m, c_m]$ must also occur?

I just can't see where the tie in is between the two sentences starting with
"For all converging subsequences ...
Then $a_n \in [a_m, c_m] \forall n \geq N_m$"

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solivas11
offline
Math Cadet
Joined: Mon, 30 Mar 2020 21:12
Posts: 7

Post subject: Re: 2.26 **Posted:** Thu, 02 Apr 2020 15:11

> **anthony.d.dennis23 wrote:**
> Are you saying that because $a_{n_k} \in [a_m, c_m]$ occurs that $a_n \in [a_m, c_m]$ must also occur?

Yes for $n \geq N$ (as per my argument, but my intuition says 'kind of').

> **anthony.d.dennis23 wrote:**
> I just can't see where the tie in is between the two sentences starting with
> "For all converging subsequences ...
> Then $a_n \in [a_m, c_m] \forall n \geq N_m$"

Since a_{n_k} converges, it is within $-\varepsilon + b < a_{n_k} < \varepsilon + b \; \forall n \geq N \in \mathbb{N}$
I tried to define ε in such a way so that $a_m < a_{n_k} < c_m \; \forall n \geq N_m$. By considering an N far out enough so to constrict the a_{n_k} $to\ b$, this would imply that there exists infinitely many elements of the sequence surrounding the limit.

Via Bolzano-Weierstrass, we know bounded sequences have at minimum one convergent subsequence. In this case, we have several convergent subsequences that just so happen to converge to the same limit. The things I'm struggling with are the subsequences which do not converge and how those apply to the original sequence.

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anthony.d.dennis23 **Post subject:** Re: 2.26 **Posted:** Thu, 02 Apr 2020 17:12

offline
Member

Joined: Sat, 28 Mar 2020 15:16
Posts: 14

I tried contradiction for this try.

$Suppose(\mathrm{a}_n)$ is bounded, and Assume that (a_n) doesn't converge, then $\exists \epsilon > 0$ s.t. $\exists N \in \mathbf{N}$, with $n \geq N$, $|a_n - a| \geq \epsilon$.
Then by Task 2.25 there is a subsequence(a_{n_k}) of (a_n) such that $|a_{n_k} - a| \geq \epsilon$ $\forall k \in \mathbf{N}$.

Since (a_n) is bounded, its subsequence (a_{n_k}) must be bounded. Then by Bolzano-Weierstrass Theorem 2.23 the bounded sequence (a_{n_k}) has a convergent subsequence $(a_{n_{k_1}}) \to a$.

But we saw previously by 2.25 that if (a_n) doesn't converge then (a_{n_k}) doesn't converge then $(a_{n_{k_1}})$ shouldn't converge.

Therefore we have a contradiction in the initial assumption that (a_n) doesn't converge.

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solivas11 **Post subject:** Re: 2.26 **Posted:** Thu, 02 Apr 2020 17:47

offline
Math Cadet

Joined: Mon, 30 Mar 2020 21:12
Posts: 7

> **anthony.d.dennis23 wrote:**
>
> I tried contradiction for this try.
>
> $Butwesawpreviouslyby2.25thatif(\mathrm{a}_n)$ doesn't converge then (a_{n_k}) doesn't converge then $(a_{n_{k_1}})$ shouldn't converge.
>
> Therefore we have a contradiction in the initial assumption that (a_n) doesn't converge.

You mean (a_n) doesn't converge in general or doesn't converge to a specifically?

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helmut

Post subject: Re: 2.26

Posted: Sun, 05 Apr 2020 16:10

online

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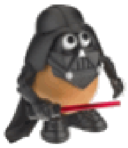

Joined: Sat, 26 Apr 2003 15:14
Posts: 2232
Location: El Paso TX (USA)

> **anthony.d.dennis23 wrote:**
>
> I tried contradiction for this try.
>
> $Suppose(a_n)$ is bounded, and Assume that (a_n) doesn't converge, then $\exists \epsilon > 0$ s.t. $\exists N \in \mathbf{N}$, with $n \geq N$, $|a_n - a| \geq \epsilon$.
> Then by Task 2.25 there is a subsequence(a_{n_k}) of (a_n) such that $|a_{n_k} - a| \geq \epsilon$ $\forall k \in \mathbf{N}$.
> Since (a_n) is bounded, its subsequence (a_{n_k}) must be bounded. Then by Bolzano-Weierstrass Theorem 2.23 the bounded sequence (a_{n_k}) has a convergent subsequence $(a_{n_{k_1}}) \to a$.
>
> But we saw previously by 2.25 that if (a_n) doesn't converge then (a_{n_k}) doesn't converge then $(a_{n_{k_1}})$ shouldn't converge.
>
> Therefore we have a contradiction in the initial assumption that (a_n) doesn't converge.

Change the order and rewrite:

First use BW to get a converging subsequence with limit a.
Then assume the original sequence does not converge to a.
Finally use 2.25.

____
The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

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anthony.d.dennis23

Post subject: Re: 2.26

Posted: Tue, 07 Apr 2020 10:15

offline

Member

Joined: Sat, 28 Mar 2020 15:16
Posts: 14

$Itisgiventhat(a_n)$ is bounded, then its subsequence (a_{n_k}) must also be bounded. Thus by Bolzano-Weierstrass Theorem 2.23 the bounded sequence (a_{n_k}) has a convergent subsequence. Call it $(a_{n_{k_1}})$ which converges to a.

Assume that (a_n) doesn't converge to a, then $\exists \epsilon > 0$ s.t. $\exists N \in \mathbf{N}$, with $n \geq N$, $|a_n - a| \geq \epsilon$.
Then by Task 2.25 there is a subsequence(a_{n_k}) of (a_n) such that $|a_{n_k} - a| \geq \epsilon$ $\forall k \in \mathbf{N}$.

Again if (a_{n_k}) doesn't converge to a, then its subsequence $(a_{n_{k_1}})$ doesn't converge to a, but we saw earlier that $(a_{n_{k_1}})$ converges to a, so this is a contradiction to the original assumption. Therefore (a_{n_k}) converges to a, and so must (a_n) also converge to a.

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helmut

Post subject: Re: 2.26

Posted: Tue, 07 Apr 2020 14:23

online

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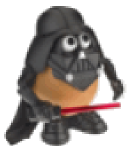

Nearly there. The last paragraph needs some work. Is (a_n_k1) really a subsequence of (a_n_k)?

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

Joined: Sat, 26 Apr 2003 15:14
Posts: 2232
Location: El Paso TX (USA)

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anthony.d.dennis23

Post subject: Re: 2.26

Posted: Thu, 09 Apr 2020 13:19

offline

Member

Joined: Sat, 28 Mar 2020 15:16
Posts: 14

$Itisgiventhat(a_n)$ is bounded, then its subsequence (a_{n_k}) must also be bounded. Thus by Bolzano-Weierstrass Theorem 2.23 the bounded sequence (a_{n_k}) has a convergent subsequence. Call it $(a_{n_{k_1}})$ which converges to a.

Assume that (a_n) doesn't converge to a, then $\exists \epsilon > 0$ s.t. $\exists N \in \mathbf{N}$, with $n \geq N, |a_n - a| \geq \epsilon$.
Then by Task 2.25 there is a subsequence(a_{n_k}) of (a_n) such that $|a_{n_k} - a| \geq \epsilon$ $\forall k \in \mathbf{N}$.

> **helmut wrote:**
>
> Nearly there. The last paragraph needs some work. Is (a_n_k1) really a subsequence of (a_n_k)?

$Nowif(a_{n_k})$ doesn't converge by the previous statement then by 2.25 there is a subsequence $(a_{n_{k_1}})$ of (a_{n_k}) such that $|a_{n_{k_1}} - a| \geq \epsilon \ \forall k_1 \in \mathbf{N}$.
But we saw by Bolzano-Weierstrass that to $(a_{n_{k_1}})$ converges to a, thus we have a contradiction to the fact that, if (a_{n_k}) doesn't converge to a then $(a_{n_{k_1}})$ doesn't converges to a . Therefore (a_{n_k}) must converges to a, which again leads to a contradiction that (a_n) doesn't converge to a. Therefore (a_n) must converge to a.

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Jocelyne Perez

Post subject: Re: 2.26

Posted: Thu, 09 Apr 2020 17:39

offline

Member

Joined: Tue, 31 Mar 2020 15:27
Posts: 10

Thank you for the correction, now I understand why it leads to contradiction.

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helmut

Post subject: Re: 2.26

Posted: Mon, 13 Apr 2020 12:11

online

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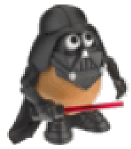

Joined: Sat, 26 Apr 2003 15:14
Posts: 2232
Location: El Paso TX (USA)

Nice. Done. Credit for Anthony.

____
The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

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