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2.32
Moderator: helmut [Moderator Control Panel]

Author	Message

violeta guzman **Post subject:** Re: 2.32 **Posted:** Thu, 16 Apr 2020 18:46

offline
Member

Joined: Tue, 31 Mar 2020 15:36
Posts: 13

Hi Anthony.
Am I right in saying that you are not proving by contradiction anymore? I noticed that you started your proof by establishing S_n does not equal x. Just wondering, why did you abandone the proof by contradition, it looks like you were making some headway.

anthony.d.dennis23 **Post subject:** Re: 2.32 **Posted:** Thu, 16 Apr 2020 21:58

offline
Member

Joined: Sat, 28 Mar 2020 15:16
Posts: 25

Violeta- It only worked if $\epsilon = |S_n - x|$. If ϵ was greater than that distance it didn't matter if $S_n = x$ or not.

Abigail- It looked so good when I wrote it on my chalkboard but I made a logical leap between these two sentences that I can't think how to fix right now.

> **anthony.d.dennis23 wrote:**
> So this leads to $|S_n - x| < \epsilon_0, \forall \epsilon > 0$.
> If we choose $\epsilon_1 > \epsilon_0$, then $\exists N_1 \in \mathbf{N} s.t. \forall n \in \mathbf{N}$ with $n \geq N_1, |S_n - x| < \epsilon_1$.

helmut **Post subject:** Re: 2.32 **Posted:** Sun, 19 Apr 2020 21:18

online

Site Admin

Joined: Sat, 26 Apr 2003 15:14
Posts: 2272
Location: El Paso TX (USA)

> **anthony.d.dennis23 wrote:**
>
> $\Leftarrow$(Trying to show x is an accumulation point of the set S)
> Let every neighborhood of x contain an element of S, call it $S_n, n \in \mathbf{N}$, that is not equal to x,...

I don't understand this sentence. Is "every neighborhood" indexed by some n?

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

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anthony.d.dennis23 **Post subject:** Re: 2.32 **Posted:** Thu, 30 Apr 2020 14:03

offline

Member

Joined: Sat, 28 Mar 2020 15:16
Posts: 25

Dr. Knaust,
This whole opening is a mess. S_n is actually only one point but I tricked myself by writing it like that into thinking it was a sequence, which is what I want to show/prove. I'll have to do some more work on it.

> **anthony.d.dennis23 wrote:**
>
> $\Leftarrow$(Trying to show x is an accumulation point of the set S)
>
> Let every neighborhood of x contain an element of S, call it $S_n, n \in \mathbf{N}$, that is not equal to x, $S_n \neq x$. So for$\epsilon > 0, S_n \neq x \in (x - \epsilon, x + \epsilon)$. So this leads to $|S_n - x| < \epsilon_0, \forall \epsilon > 0$.

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helmut **Post subject:** Re: 2.32 **Posted:** Thu, 30 Apr 2020 15:50

online

Site Admin

Joined: Sat, 26 Apr 2003 15:14
Posts: 2272
Location: El Paso TX (USA)

Don't you want to make the ϵ's smaller and smaller?

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

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solivas11 **Post subject:** Re: 2.32 **Posted:** Thu, 30 Apr 2020 22:39

offline

Member

Joined: Mon, 30 Mar 2020 21:12
Posts: 20

I thought it would be a good idea to start back at square one. Dr. Knaust gave some pointers on how to modify the original plan by Maria.

> **Mdluevano2 wrote:**
>
> The real number x is an accumulation point of set S if and only if every neighborhood of x contains an element of S different from x.
>
> Show x is an accumulation point of S
> $\Rightarrow$
> Let ε_1 =1
> Then we can find $x_1 \in (x+\varepsilon_1, x-\varepsilon_1)\ of\ \ set\ S\ where\ x_1 \neq x$
>
> Let ε_2 = $|x_1 - x|$
> Since $x_1 \neq x \Rightarrow \epsilon_2 > 0$
> Then we can find
> $x_2 \in (x+\varepsilon_2, x-\varepsilon_2)\ of\ \ set\ S\ where\ x_2 \neq x\ or\ x_2 \neq x_1$
>
> Let ε_3 = $|x_2 - x|$
> Since $x_2 \neq x \Rightarrow \epsilon_3 > 0$
> Then we can find
> $x_3 \in (x+\varepsilon_3, x-\varepsilon_3)\ of\ \ set\ S\ where\ x_3 \neq x\ or\ x_i \neq x_j\ for\ i \neq j$
>
> If we continue
> Then we can find
> $x_n \in (x+\varepsilon_n, x-\varepsilon_n)\ of\ \ set\ S\ where\ x_n \neq x\ or\ x_i \neq x_j\ for\ i \neq j$

> **Mdluevano2 wrote:**
>
> Neighborhood of x contains an element of S different from x
> $\Leftarrow$
> If $x_n \in$ (x+1, x-1) of set S
> Then we can find $x_n \neq x \forall n \geq N$ which are different from S

I think a similar, if not same, procedure can be used for the other direction. Since x is an accumulation point, then you can vary the epsilon to find unique elements of S in the neighborhood of x. Alternatively, if you can vary the epsilon so as to find unique elements of S in the neighborhood of x, and do this for infinitely many epsilon (since $x_n \neq x$), then infinitely many elements of S can be found in the neighborhood of x, making it an accumulation point.

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helmut | **Post subject:** Re: 2.32 | **Posted:** Sun, 03 May 2020 12:29

online

Site Admin

Idea is sound.
How do you make sure that $\lim_{n\to\infty} \varepsilon_n = 0$?

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

Joined: Sat, 26 Apr 2003 15:14
Posts: 2272
Location: El Paso TX (USA)

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solivas11

offline

Member

Joined: Mon, 30 Mar 2020 21:12
Posts: 20

Post subject: Re: 2.32 **Posted:** Wed, 06 May 2020 18:59

The real number x is an accumulation point of set S if and only if every neighborhood of x contains an element of S different from x.

> **solivas11 wrote:**
>
> Since x is an accumulation point, then you can vary the epsilon to find unique elements of S in the neighborhood of x.

> **Mdluevano2 wrote:**
>
> The real number x is an accumulation point of set $S \Rightarrow$ every neighborhood of x contains an element of S different from x.
>
> Let ε_1 =1
> Then we can find $x_1 \in (x+\varepsilon_1, x-\varepsilon_1)\ of\ set\ S\ where\ x_1 \neq x$
>
> Let ε_2 = $|x_1 - x|$
> Since $x_1 \neq x \Rightarrow \epsilon_2 > 0$
> Then we can find
> $x_2 \in (x+\varepsilon_2, x-\varepsilon_2)\ of\ set\ S\ where\ x_2 \neq x\ or\ x_2 \neq x_1$
>
> Let ε_3 = $|x_2 - x|$
> Since $x_2 \neq x \Rightarrow \epsilon_3 > 0$
> Then we can find
> $x_3 \in (x+\varepsilon_3, x-\varepsilon_3)\ of\ set\ S\ where\ x_3 \neq x\ or\ x_i \neq x_j\ for\ i \neq j$
>
> If we continue
> Then we can find
> $x_n \in (x+\varepsilon_n, x-\varepsilon_n)\ of\ set\ S\ where\ x_n \neq x\ or\ x_i \neq x_j\ for\ i \neq j$

> **solivas11 wrote:**
>
> Alternatively, if you can vary the epsilon so as to find unique elements of S in the neighborhood of x, and do this for infinitely many epsilon (since $x_n \neq x$), then infinitely many elements of S can be found in the neighborhood of x, making it an accumulation point.

> **Mdluevano2 wrote:**
>
> Every neighborhood of x contains an element of S different from x $\Rightarrow$ The real number x is an accumulation point of set S
>
> Let ε_1 =1
> Then we can find $x_1 \in (x+\varepsilon_1, x-\varepsilon_1)\ of\ set\ S\ where\ x_1 \neq x$
>
> Let ε_2 = $\dfrac{|x_1 - x|}{2}$
>
> Since $x_1 \neq x \Rightarrow \epsilon_2 > 0$
> Then we can find
> $x_2 \in (x+\varepsilon_2, x-\varepsilon_2)\ of\ set\ S\ where\ x_2 \neq x\ or\ x_2 \neq x_1$
>
> Let ε_3 = $\dfrac{|x_2 - x|}{2}$
>
> Since $x_2 \neq x \Rightarrow \epsilon_3 > 0$
> Then we can find
> $x_3 \in (x+\varepsilon_3, x-\varepsilon_3)\ of\ set\ S\ where\ x_3 \neq x\ or\ x_i \neq x_j\ for\ i \neq j$
>
> If we continue
> Then we can find
> $x_n \in (x+\varepsilon_n, x-\varepsilon_n)\ of\ set\ S\ where\ x_n \neq x\ or\ x_i \neq x_j\ for\ i \neq j$

> **helmut wrote:**
>
> Idea is sound. :)
> How do you make sure that $\lim_{n\to\infty} \varepsilon_n = 0$?

Since we bisect the $\varepsilon\ s.t.\ \varepsilon_{n+1} = \dfrac{|x_n - x|}{2}$, then we decrease the ε exponentially.

We can do this since every neighborhood of x contains an element of S different from $x \iff \forall x_n \in S\ x_n \neq x \Rightarrow \forall n \in \mathbb{N}\ \varepsilon_{n+1} > 0$, including the original ε_1 initially set.

Thus, $\lim_{n\to\infty} \varepsilon_n = 0$ and for infinitely many epsilon (since $x_n \neq x$), then infinitely many elements of S can be found in the neighborhood of x, making it an accumulation point.

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helmut | **Post subject:** Re: 2.32 | **Posted:** Thu, 07 May 2020 13:49

online

Site Admin

Joined: Sat, 26 Apr 2003 15:14
Posts: 2272
Location: El Paso TX (USA)

Nice. I credit for Sergio!

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

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