S.O.S. Mathematics CyberBoard

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3.2

Moderator: helmut [Moderator Control Panel]

Author	**Message**

anthony.d.dennis23

offline

Member

Joined: Sat, 28 Mar 2020 15:16
Posts: 25

Post subject: 3.2 **Posted:** Thu, 30 Apr 2020 13:49

Let $D \subset \mathbf{R}$, let $f : D \to \mathbf{R}$ be a continuous function and let x_0 be an accumulation point of D. Then the following are equivalent:

$1)$ $\lim_{x \to x_0} f(x)$ exists and is equal to L.

$\mathbf{2})$ Let (x_n) be any sequence of elements in D that converge to x_0, and satisfies that $x_n \neq x_0$ for all $n \in \mathbf{N}$. Then the sequence $f(x_n)$ converges to L.

SOLUTION
$1 \Rightarrow 2$
Let $D \subset \mathbf{R}$, let $f : D \to \mathbf{R}$. Also let $\lim_{x \to x_0} f(x) = L$. Assume $x_n \in D$ is a sequence that converges to x_0.
Since $\lim_{x \to x_0} f(x) = L$ then by definition $\forall \epsilon, \exists \delta > 0 s.t. |f(x) - L| < \epsilon$ whenever $0 < |x - x_0| < \delta$. We also know that $x_n \to x_0$. So, by convergence definition, for $\delta > 0, \exists N \in \mathbf{N} s.t. |x_n - x_0| < \delta, \forall n \geq N.$

Since we need to show that the sequence $f(x_n)$ converges to L. Choose $\epsilon > 0$, then for $\forall n \geq N \text{ and } \delta > 0$ whenever $0 < |x_n - x_0| < \delta$ we have $|f(x_n) - L| < \epsilon$.

helmut

online

Site Admin

Post subject: Re: 3.2 **Posted:** Thu, 30 Apr 2020 16:43

1->2 correct. That was a problem on Test 2.

We still need 2->1.

$2 \times 1/2$ credit for Anthony for this half and one half of 2.35.

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

Joined: Sat, 26 Apr 2003 15:14
Posts: 2274
Location: El Paso TX (USA)

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solivas11 offline Member

Post subject: Re: 3.2 **Posted:** Sat, 02 May 2020 19:58

I initially misunderstood 2) as "Then the sequence f(x) cv to L," instead of "Then the sequence f(x_n) cv to L." Thank you for the clarification, Anthony.

Joined: Mon, 30 Mar 2020 21:12
Posts: 20

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helmut online Site Admin

Post subject: Re: 3.2 **Posted:** Sun, 03 May 2020 12:50

We still need 2->1.

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

Joined: Sat, 26 Apr 2003 15:14
Posts: 2274
Location: El Paso TX (USA)

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solivas11 offline Member

Post subject: Re: 3.2 **Posted:** Sun, 03 May 2020 14:31

> **anthony.d.dennis23 wrote:**
>
> Let $D \subset \mathbf{R}$, let $f : D \rightarrow \mathbf{R}$ be a continuous function and let x_0 be an accumulation point of D. Then the following are equivalent:
>
> $\mathbf{1})$ $\lim_{x \rightarrow x_0} f(x)$ exists and is equal to L.
>
> $\mathbf{2})$ Let (x_n) be any sequence of elements in D that converge to x_0, and satisfies that $x_n \neq x_0$ for all $n \in \mathbf{N}$. Then the sequence $f(x_n)$ converges to L.

Joined: Mon, 30 Mar 2020 21:12
Posts: 20

My attempt at $2 \Rightarrow 1$

2) $x_n \in D, \ \forall n \in \mathbb{N} \ x_n \neq x_0, \ x_n \rightarrow x_0 \Rightarrow f(x_n) \rightarrow \ L$

$\iff$ (Expanding the above)

$x_n \in D, \ \forall n \in \mathbb{N} \ x_n \neq x_0,$
$\forall \delta > 0 \ \exists N_1 \in \mathbb{N} \ s.t. \ \forall n \geq N_1 |x_n - x_0| < \delta$
$\Rightarrow \forall \varepsilon > 0 \ \exists N_2 \in \mathbb{N} \ s.t. \ \forall n \geq N_2 |f(x_n) - L| < \varepsilon$

Since $\forall n \in \mathbb{N} \ x_n \neq x_0$, we have $0 < |x_n - x_0| < \delta$

Let $N = max\{N_1, N_2\}$
Now,
$\forall n \geq N, \text{ whenever } 0 < |x_n - x_0| < \delta \text{ is true, then } |f(x_n) - L| < \varepsilon \text{ is true}$

$\Longleftrightarrow$ (rephrasing the above)

$\forall \epsilon > 0, \exists \delta > 0 \ s.t. \ |f(x) - L| < \epsilon \text{ whenever } 0 < |x - x_0| < \delta \text{ true}$
$\Longleftrightarrow$
$\lim_{x \to x_0} f(x) = L$

I think this is almost there. Just need a bit of help from jumping from x_n to just x

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anthony.d.dennis23 **Post subject:** Re: 3.2 **Posted:** Mon, 04 May 2020 18:21

offline
Member

Joined: Sat, 28 Mar 2020 15:16
Posts: 25

> **helmut wrote:**
> 1->2 correct. That was a problem on Test 2.

Well $%*t, I guess I should have spent some more time on this a couple of weeks ago.

> **solivas11 wrote:**
> I think this is almost there. Just need a bit of help from jumping from x_n to just x

Sergio,
You're stuck in the same place I was. The definitions don't lend to an easy transition from x_n to x. I am going to work on one of the cons, hopefully have something by tomorrow.

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solivas11 **Post subject:** Re: 3.2 **Posted:** Tue, 05 May 2020 15:00

offline
Member

Joined: Mon, 30 Mar 2020 21:12
Posts: 20

> **anthony.d.dennis23 wrote:**
>
> > **helmut wrote:**
> > 1->2 correct. That was a problem on Test 2.
>
> Well $%*t, I guess I should have spent some more time on this a couple of weeks ago.

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anthony.d.dennis23

offline

Member

Joined: Sat, 28 Mar 2020 15:16
Posts: 25

Post subject: Re: 3.2 **Posted:** Wed, 06 May 2020 21:15

1 $\Leftarrow$ 2
Let $D \subset \mathbf{R}$, let $f : D \to \mathbf{R}$ be a continuous function and let x_0 be an accumulation point of D.
Let (x_n) be any sequence of elements in D that converge to x_0, and satisfies that $x_n \neq x_0$ for all $n \in \mathbf{N}$. Then the sequence $f(x_n)$ converges to L.$(\mathbf{P})$
$\lim_{x \to x_0} f(x)$ exists and is equal to L.$(\mathbf{Q})$

Assume $\lim_{x \to x_0} f(x) D.N.E$ then $\exists \epsilon > 0, s.t. \forall \delta > 0$ we have $0 < |x - x_0| < \delta$ and $|f(x) - L| \geq \epsilon$.
Choose $\delta = 1$ then we can find an x_1 to satisfy $0 < |x_1 - x_0| < \delta$ and thus $|f(x_1) - L| \geq \epsilon$.
Choose $\delta = \frac{1}{2}$ then we can find an x_2 to satisfy $0 < |x_2 - x_0| < \delta$ and thus $|f(x_2) - L| \geq \epsilon$.
...
Choose $\delta = \frac{1}{N}$ then we can find an x_N to satisfy $0 < |x_N - x_0| < \delta$ and thus $|f(x_N) - L| \geq \epsilon$.
Thus for $n \geq N$ we have x_n to satisfy $0 < |x_n - x_0| < \delta$ and thus $|f(x_n) - L| \geq \epsilon$ that is $f(x_n)$ does not converge to L.
So we have $\mathbf{not Q} \Rightarrow \mathbf{not P}$ thus by contrapositive $\mathbf{P} \Rightarrow \mathbf{Q}$

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helmut

online

Site Admin

Joined: Sat, 26 Apr 2003 15:14
Posts: 2274
Location: El Paso TX (USA)

Post subject: Re: 3.2 **Posted:** Thu, 07 May 2020 13:57

Correct. 1 credit for Anthony.

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

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