# S.O.S. Mathematics CyberBoard

Your Resource for mathematics help on the web!



## 3.6

**Moderator:** helmut [ Moderator Control Panel ]



**Author** **Message**

**helmut**

online

Site Admin

**Joined:** Sat, 26 Apr 2003 15:14
**Posts:** 2284
**Location:** El Paso TX (USA)

**Post subject:** Re: 3.6 **Posted:** Thu, 07 May 2020 13:47

Getting there.

Case 1: Isn't $f(x) = 1$, if $x$ is rational? Also: How do you know $L > 1/2$, if you only assume $L \neq 0$?

---

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

Top profile pm email edit quote

**solivas11**

offline

Member

**Joined:** Mon, 30 Mar 2020 21:12
**Posts:** 24

**Post subject:** Re: 3.6 **Posted:** Thu, 07 May 2020 23:21

Let $f : (0, 1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

For which values of $x_0$ does $f(x)$ have a limit at $x_0$? What is the limit?

Based on some feedback from office hours. Based off the definition of $f$, then $f(x)$ is either 1 or 0 based on whether it is rational or irrational.

Assume that the limit exists.
Then $\exists c \in \{0, 1\}$ $s.t.$ $\lim_{x \to x_0} f(x) = c$

By the definition of the limit (and borrowing Estefany's work from 3.3, and Abigail and Jocelyne's work from 4.6), we say

$\forall \varepsilon > 0,\ \exists \delta > 0\ such\ that\ |f(x) - c| < \varepsilon$
$whenever\ x \in (x_0 - \delta, x_0 + \delta) \cap (0,1] \setminus \{x_0\}$
$and\ 0 < |x - x_0| < \delta$

Let $\varepsilon = \frac{1}{2},\ \delta > 0$

Using what we know about the rational and irrational numbers being dense in $\mathbb{R}$ from 1.10 and 1.11, respectively,
$\forall a, b \in \mathbb{R}\ \exists x \in \mathbb{Q}\ s.t.\ a < x < b$
$\forall a, b \in \mathbb{R}\ \exists y \in \mathbb{R} \setminus \mathbb{Q}\ s.t.\ a < y < b$

Since $(x_0 \pm \delta) \in \mathbb{R}$
Then $\exists x \in (x_0 - \delta, x_0 + \delta) \cap (0,1] \setminus \{x_0\}\ s.t.\ f(x) = 1$
and $\exists y \in (x_0 - \delta, x_0 + \delta) \cap (0,1] \setminus \{x_0\}\ s.t.\ f(y) = 0$

Now for
$x, y \in (x_0 - \delta, x_0 + \delta) \cap (0,1] \setminus \{x_0\}$
$such\ that\ |x - x_0| < \delta\ and\ |y - x_0| < \delta$

Then it should be the case $|f(x) - f(y)| < \varepsilon\ when\ |x - y| < \delta$

Yet, for our given $x, y$ values,

$$|f(x) - f(y)| < \varepsilon \iff |1 - 0| < \tfrac{1}{2} \iff 1 < \tfrac{1}{2}$$

which is false. Thus $|f(x) - f(y)| \geq \varepsilon$

This contradicts the given hypothesis, $\exists c \in \{0,1\}\ s.t.\ \lim_{x \to x_0} f(x) = c$, therefore,
$\lim_{x \to x_0} f(x)$ does not exists.

Top | profile | pm | email | edit | quote

**helmut**

online

Site Admin

**Joined:** Sat, 26 Apr 2003 15:14
**Posts:** 2284
**Location:** El Paso TX (USA)

**Post subject:** Re: 3.6

**Posted:** Fri, 08 May 2020 12:18

Convoluted, but correct. Credit for Sergio.

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

Top | profile | pm | email | edit | quote

Display posts from previous: All posts ▼ Sort by Post time ▼ Ascending ▼ Go

newtopic locked **Page 2 of 2** [ 18 posts ] **Go to page Previous 1, 2**

**Board index » Math 3341 » Chapter 3** All times are UTC - 6 hours

**Who is online**

Users browsing this forum: **helmut**

Quick-mod tools: Unlock topic Go

You **can** post new topics in this forum
You **can** reply to topics in this forum
You **can** edit your posts in this forum
You **can** delete your posts in this forum

Search for: Go Jump to: Chapter 3 Go

[ Administration Control Panel ]