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HW: 4.1.1(1,3,4 a-c,8cd), 4.1.2(1-3,7,8,11)

1. According to the set definition of ordered pair, what (b,a) ?

Def: ordered pairs (a,b) as the set $\{\{a\}, \{a,b\}\}$
 It is in terms of the ordered pair (a,b) .
 Therefore, $(b,a) = \{\{b\}, \{b,a\}\}$

3. Use the definition of equality of sets and equality of ordered pairs to prove that $(a,b) = (c,d) \iff a=c$ and $b=d$.

Def of set $(a,b) = \{\{a\}, \{a,b\}\}$
 $(c,d) = \{\{c\}, \{c,d\}\}$
 First consider $(a,b) = (c,d)$ then
 $\{\{a\}, \{a,b\}\} = \{\{c\}, \{c,d\}\}$
 Equality of sets $\{a\} = \{c\}$ and $\{a,b\} = \{c,d\}$
 $a=c \dots b=d$
 Therefore, if $(a,b) = (c,d)$ then $a=c$ and $b=d$
 then $\{a\} = \{c\}$ and $\{a,b\} = \{c,d\}$
 $\{\{a\}, \{a,b\}\} = \{\{c\}, \{c,d\}\}$
 $(a,b) = (c,d)$
 Therefore, $(a,b) = (c,d) \iff a=c$ and $b=d$.

4. a. reflexive and symmetric but not transitive.
 $R = \{(1,1), (2,2), (3,3), (0,2), (4,3), (2,1), (3,1)\}$

1. $(1,1), (2,2), (3,3) \in R$, it is reflexive
 2. $(1,2), (2,1) \in R$ & $(1,3), (3,1) \in R$, it is symmetric
 3. R is not transitive as $(3,1), (1,2) \in R$ but $(3,2) \notin R = (a,d)$

b. symmetric and transitive but not reflexive.
 let $A = \{-5, -6\}$
 Relation R on A
 $R = \{(-5, -6), (-6, -5), (-5, -5)\}$
 1. not reflexive as $(-6, -6) \notin R$
 2. it is symmetric as $(-5, -6) \in R$ and $(-6, -5) \in R$
 3. It is transitive as $(-5, -6), (-6, -5) \in R$ also $(-5, -5) \in R$

c. reflexive and transitive but not symmetric
 let $A = \{a, b, c, d\}$
 $R = \{(a,a), (a,b), (a,c), (b,b), (c,c), (c,d), (d,d)\}$
 1. reflexive because $(a,a), (b,b), (c,c),$ and (d,d)
 2. Not symmetric because $(a,b) \in R$ and $(b,a) \notin R$
 3. transitive because $(a,b) \in R$ and $(b,c) \in R$, while $(a,c) \in R$.

2. $\sin y = \cos y \cdot \tan y$ \exists true
 \forall false

$y = 0$
 $\sin(0) = \cos(0) \cdot \tan(0)$
 $0 = 0$

$y = \frac{\pi}{2}$
 $\sin(\frac{\pi}{2}) = \cos(\frac{\pi}{2}) \cdot \tan(\frac{\pi}{2})$
 $1 \neq 0 \cdot \text{Undefined}$

8. a) $3A = 5B$
 b) $3B = 5A$
 c) $\frac{A}{3} = \frac{5}{5}$
 d) $5A = 3B$

Equivalent equations are those that have exactly same equations.
 b and d are equal
 That is the equations $3B = 5A$, $5A = 3B$ are equivalent
 For, $x=y$ is equivalent to saying that $y=x$
 But equation a and c are not equal
 Since $3A = 5B$ can be written as $\frac{A}{B} = \frac{5}{3}$

8a) Let $(a,b), (c,d)$ and $(e,f) \in S$
 $(a,b) \times (c,d) = (ac + bd, ad + bc)$
 $(a,b) \times (e,f) = (ae + bf, af + be)$
 $(a,b) \times (c,d) + (a,b) \times (e,f) = (ac + bd + ae + bf, ad + bc + af + be)$

On the other hand $(c,d) + (e,f) = (c+e, d+f)$
 And $(a,b) \times ((c,d) + (e,f)) = (a,b) \times (c+e, d+f)$
 $= (ac + ae + bd + bf, ad + af + bc + be)$
 $= (a,b) \times (c,d) + (a,b) \times (e,f)$

8b) Define $f: S \rightarrow Z$ by $f(a,b) = a-b$
 f is one-to-one:
 Let $(a,b), (c,d) \in S$ such that $f(a,b) = f(c,d)$
 $\Rightarrow a-b = c-d$
 $\Rightarrow a+c = b+d$
 Therefore (a,b) and (c,d) are equivalent
 Hence f is one-to-one
 f is onto:
 Let $x \in Z$ be any element
 Case (i): If $x < 0$, then clearly $-x+1, -2x+1 > 0$
 Take $a = -x+1, b = -2x+1$
 Then $f(a,b) = a-b = x$
 Therefore f is onto
 Case (ii): Trivial
 Let $(a,b), (c,d) \in S$. Then $f(a,b) = a-b$ and $f(c,d) = c-d$
 Therefore $f(a,b) \cdot f(c,d) = (a-b) \cdot (c-d) = (ac + bd) - (bc + ad)$
 On the other hand
 $f((a,b) \times (c,d)) = f(ac + bd, bc + ad) = ac + bd - bc - ad$
 $f((a,b) + (c,d)) = f(a+c, b+d) = (a+c) - (b+d) = (a-b) + (c-d)$
 $= f(a,b) + f(c,d)$
 Therefore f is homomorphism
 Hence $(S, +, \times)$ is isomorphism to $(Z, +, \cdot)$.

7. Which two equations are equivalent?

Explain.

a. $|x| = -5$ b. $x = 5$ c. $|x| = 5$ d. $x = |5|$

b and d are equivalent

$x = |5|$ always gives $x = 5$ (absolute value)

a. $|x| = -5$ is a false statement

Absolute value cannot yield negative

so a and c are not ~~equivalent~~
equivalent

11. Write a singleton equation that describes the coordinates (x, y) of all the points that are 4 units from $(-3, 9)$ and 6 units from $(2, 5)$

think of the circle equation

$$(x-h)^2 + (y-k)^2 = r^2$$

for $(-3, 9)$ $(x - (-3))^2 + (y - 9)^2 = 4^2$

$$\Leftrightarrow (x+3)^2 + (y-9)^2 = 16$$

$$\Leftrightarrow (x+3)^2 + (y-9)^2 - 16 = 0$$

for $(2, 5)$ $(x-2)^2 + (y-5)^2 = 6^2$

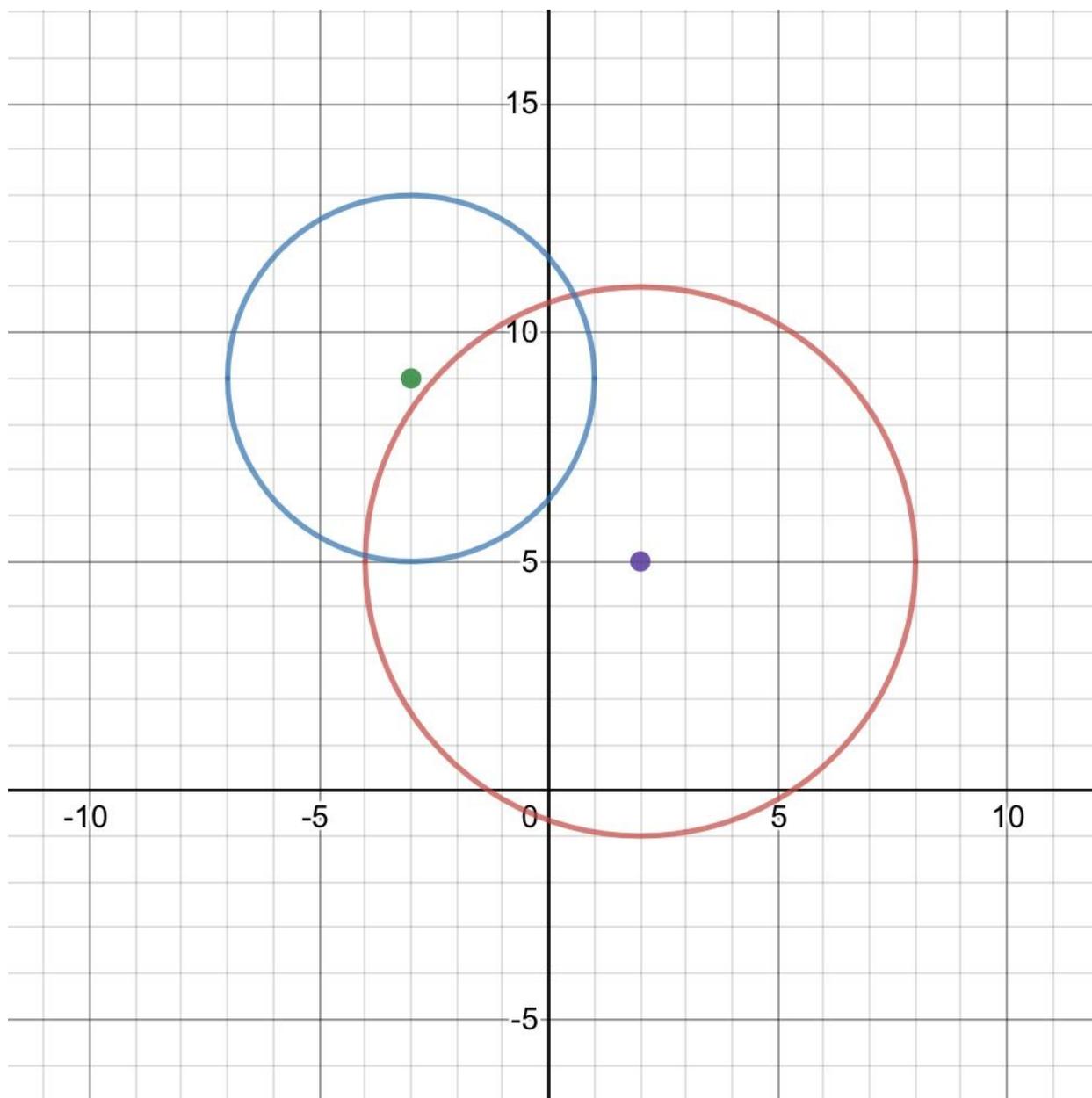
$$\Leftrightarrow (x-2)^2 + (y-5)^2 = 36$$

$$\Leftrightarrow (x-2)^2 + (y-5)^2 - 36 = 0$$

since both equations equal 0 we can

combine them into one

$$(x-2)^2 + (y-5)^2 - 36 = (x+3)^2 + (y-9)^2 - 16$$



1. $(x+10)^2 = x^2 + 100$

a. $\exists x \in \mathbb{R}, (x+10)^2 = x^2 + 100$

distribute ~~$x^2 + 20x + 100 = x^2 + 100$~~

$20x = 0$

$x = 0$

true with existential

b. $\forall x \in \mathbb{R}, (x+10)^2 = x^2 + 100$

again distribute

and simplify $20x = 0$

false with universal

3. $\frac{10+2z}{6+z} = 2$ multiply by $6+z$ on both sides

$10+2z = 2(6+z)$ distribute

$10+2z = 12+2z$ combine like terms

$0 = 2$

false

a. $\exists x \in \mathbb{R}, 0 = 2$ false

b. $\forall x \in \mathbb{R}, 0 = 2$ false