Section 4.2.3

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Leading to a Quadratic Equation

- Standard form -

$$ax^2 + bx + c = 0$$

- Ex: Find two numbers whose sum is 10 and whose product is 18.

(1) $p + q = 10 \quad \& \quad pq = 18$

(2) $18/q + q = 10$

(3) $18 + q^2 = 10q$

(4) $q^2 - 10q + 18 = 0$

Vieta's Theorem

- If p & q are solutions to $x^2 + bx + c = 0$,

  then **a**. $\boldsymbol{p + q = -b}$ and **b.** $\boldsymbol{pq = c}$

Proof: We prove the theorem for the case when p & q are distinct solutions to $x^2 + bx + c = 0$.

We have: $p^2 + bp + c = 0$ and $q^2 + bq + c = 0$

Then: $p^2 + pb + c = q^2 + qn + c$,

$p^2 - q^2 + bp - bq = 0$,

$(p - q)(p + q + b) = 0$.

So, $p = q$ or $\mathbf{p + q = -b}$.

Then, $p^2 + pq + bp = 0$ or $(p^2 + bp + c = 0)$.

Finally, $c = -p^2 - bp = p(-p - b) = \mathbf{pq = c}$.

Corollary

- p and q are two solutions to the quadratic equation ($ax^2 + bx + c = 0$) if and only if

$$p + q = -b/a \qquad , \qquad pq = c/a$$

Applying the Theorem

- Ex: Find two numbers whose sum is 10 & product is 18.

  Let $p = 5 + x$ and $q = 5 - x$

Since, **$pq = 18$**, then $(5 + x)(5 - x) = 18$,

$25 - x^2 = 18 \longrightarrow x^2 = 7 \longrightarrow x = \pm\sqrt{7}$.

- $p = 5 + \sqrt{7}$, $q = 5 - \sqrt{7}$.

- Then we see:

  $p + q$ $= 5 + \sqrt{7} + 5 - \sqrt{7} =$ **10**

  and

  pq $= (5 + \sqrt{7})(5 - \sqrt{7}) = 25 - 5\sqrt{7} + 5\sqrt{7} - 7$

  $= 25 - 7 =$ **18**.

Practice

- Find two numbers whose sum is 8 and whose product is 13.

  $p + q = 8$
  $pq = 13$

- Find two numbers whose sum is 14 and whose product is 36.

  $p + q = 14$

  $pq = 36$

- $p = 4 + \sqrt{3}$, $q = 4 - \sqrt{3}$

- $p = 7 + \sqrt{13}$, $q = 7 - \sqrt{13}$

Solving The Quadratic

Theorem 4.6 (Quadratic Formula)

a. For all real numbers *b* and *c*

$$x^2 + bx + c = 0 \Leftrightarrow x = \frac{-b}{2} \pm \sqrt{\frac{b^2}{4} - c}$$

b. For all real numbers *b* and *c*

$$x^2 + 2bx + c = 0 \Leftrightarrow x = -b \pm \sqrt{b^2 - 4c}$$

c. For all real numbers *b* and *c*, with a ≠ 0

$$a^2 + bx + c = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Keep in mind that the expressions under the radical sign may be negative this means the quadratic has non real solutions

Which part of Theorem 4.6 can you use and what are the solutions?

$x^2 + 7x + 3 = 0$

$4x^2 + 9x + 5 = 0$

$x^2 + 10x + 3 = 0$

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Solving the Quadratic cont.

Discriminant: The number inside the radical, determines how many real solutions the equation has.

If the coefficients of the quadratic equation are real numbers, then there are 2,1, or 0 real solutions accordingly as the discriminant is greater than, equal to, or less than 0. This however, does not apply when the coefficients are not real.

Discriminant > 0 , 2 real solutions

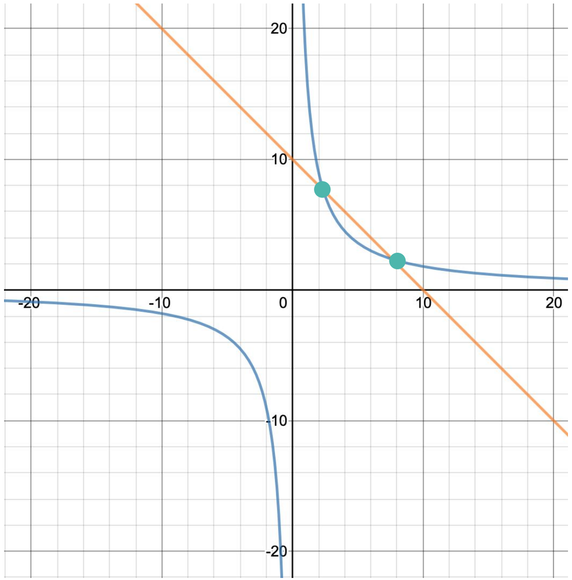
Discriminant = 0 , one real solution that is repeated

Discriminant < 0 , no real solution

A Geometric Picture

$$\begin{cases} p + q = 10 \\ pq = 18 \end{cases}$$

The intersection of the hyperbola and the line are the two solutions to the system. The graph also suggests that if values other than 10 and 18 had been chosen for the sum and product, then it is possible that there would be no real solution.



Cubic

Cubic Equation is an equation from P(x) = Q(x), where P(x) and Q(x) are polynomials of degree ≤ 3 , and one or the other has degree 3. Every cubic equation in one variable x is equivalent to an equation of the from

$$ax^3 + bx^2 + cx + d = 0$$

Cubic equation : 1, (2), or 3 solutions.

Quartic equation : 0, (1), 2, (3), or 4 real solutions.

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Cubic

Equation f(x) = 0

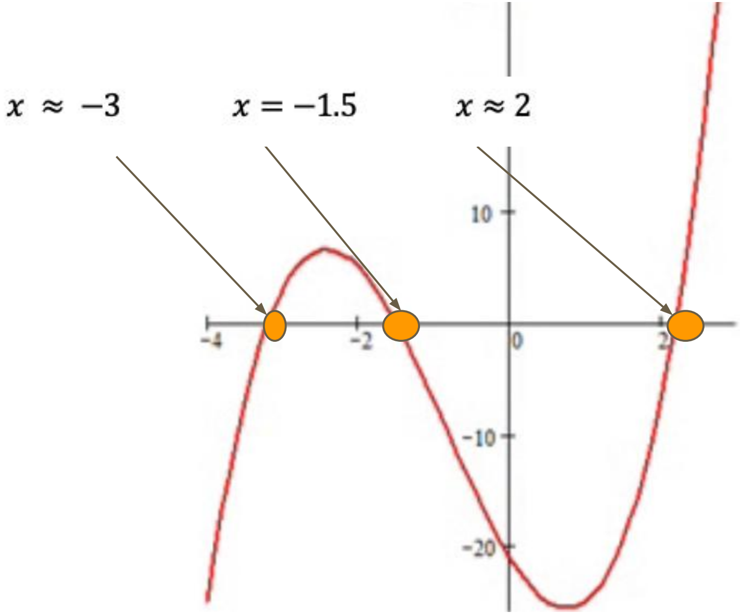
Take a good graph of the function f

X - intercepts solution

Ex)

$$2x^3 + 5x^2 - 11x - 21 = 0$$

$$f(x) = 2x^3 + 5x^2 - 11x - 21$$



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Rational Zero Theorem

If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ has integer coefficients and $\frac{p}{q}$ (where $\frac{p}{q}$ is reduced) is a rational zero, then p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

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Rational Zero Theorem

Polynomial with Integer coefficient

Candidate for rational solution:

All fractions a/b

a dividing last coefficient

b dividing first coefficient

Ex)

$$2x^3 + 5x^2 - 11x - 21 = 0$$

Divisor of -21 are (+ or -) 1, 3, 7, and 21

Divisor of 2 are (+ or -) 1 and 2

$$\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{7}{1}, \pm\frac{21}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}$$

Then check which candidate is a solution.

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Other Solutions

a Solution *(x - a)* a factor of the polynomial *x - (- 3/2) = x + 3/2* a factor

of $2x^3 + 5x^2 - 11x - 21 = 0$

$$\left(x + \frac{3}{2}\right)(J) = 2x^3 + 5x^2 - 11x - 21$$

$$(J) = \frac{2x^3 + 5x^2 - 11x - 21}{\left(x + \frac{3}{2}\right)}$$

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Long Division

$$2x^3 + 5x^2 - 11x - 21 : x + \frac{3}{2} = 2x^2 + 2x - 14$$

$$\underline{2x^3 + 3x^2}$$

$$0 + 2x^2 - 11x - 21$$

$$\underline{2x^2 + 3x}$$

$$0 \quad -14x - 21$$

$$\underline{-14x - 21}$$

$$0 + 0$$

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Remaining 2 Solutions

$$(x+\frac{3}{2})(2x^2+2x-14)=2x^3+5x^2-11x-21$$

$$(x+\frac{3}{2})2(x^2+x-7)=2x^3+5x^2-11x-21$$

Either $x+\frac{3}{2}=0,\ x=-\frac{3}{2}$

or $x^2+x-7=0$

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Quadratic Formula

Quadratic formula

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-7)}}{2} = \frac{-1 \pm \sqrt{29}}{2}$$

X = 2.193

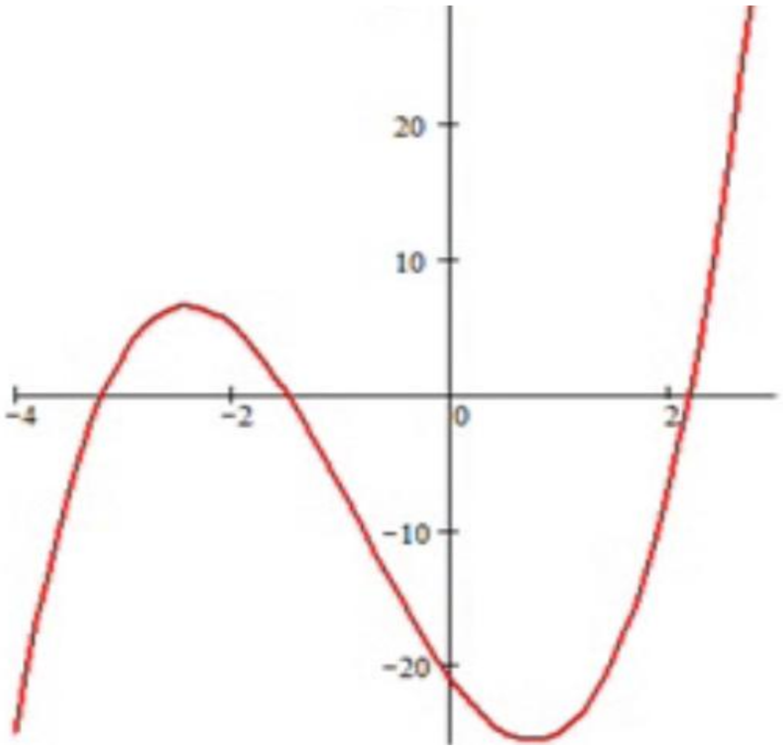
x= -3.193

Answer

X = -3/2

X = 2.193

X = -3.193



Procedure For Cubic Equation

- Find one rational solution (if there is one) a using the Rational Zero Theorem.
- Write the given polynomial as a product of $x - a$ and a quadratic polynomial, using long division.
- Find the solution of this quadratic polynomial, using quadratic formula.

(This will work if one of the solutions are Rational)

Quartic Equation

SAME METHOD

- Try to find one solution. Then factor and get a cubic equation.
- Then solve the cubic equation as before.

(This will work if two of the solutions are Rational)

Homework

1)

a. Find two numbers whose sum is -10 and whose product is 5.
b. Find two numbers whose sum is 1 and whose product is 1.
c. Find the dimensions of a rectangle whose area is

2)

Prove the corollary to Theorem 4.5.

3)

a. Show that Theorem 4.6b follows from Theorem 4.6a.
b. Show that Theorem 4.6 follows from Theorem 4.6a.

4)

Show algebraically that when the line with equation x + y =k does not intersect the hyperbola x y =18.

5)

Determine the positive difference of the solution to the quadratic equation

6)

a. If *m* and *n* are solution to , find a formula from in terms of *a, b and c.*
b. If *m* and *n* are solution to , find a formula from in terms of *a, b and c.*