Lesson 4.3.1-4.3.2 Homework Solutions

Tuesday, April 14, 2020 3:44 PM

4.3.1 Problems

2. (a) $x+\sqrt{2-x} = 4+\sqrt{2-x}$

Let $f(x) = x+\sqrt{2-x}$ & $g(x) = 4+\sqrt{2-x}$.
Let $h(x) = -\sqrt{2-x}$ be the quantity we want to add to both sides.
The intersection of the domains of f, g, & h is $\{x : x \leq 2\}$. (Radicand needs to be ≥ 0).
Now, consider

$$f(x)+h(x) = g(x) = h(x)$$
$$\Rightarrow x+\sqrt{2-x}-\sqrt{2-x} = 4+\sqrt{2-x}-\sqrt{2-x}$$
$$\Rightarrow x = 4$$

Since $x=4$ is not in the intersection of the domains, there is no solution to the equation.

(b) $y^2 + 6\log(3-2y) = 25 + 6\log(3-2y)$

Let $f(y) = y^2 + 6\log(3-2y)$ & $g(y) = 25 + 6\log(3-2y)$.
Let $h(y) = -6\log(3-2y)$ be the quantity we want to add to both sides.
The intersection of the domains of f, g, & h is $\{y : y < 3/2\}$ (Quantity inside log has to be >0.)
Now, consider

$$f(y)+h(y) = g(y)+h(y)$$
$$\Rightarrow y^2+6\log(3-2y)-6\log(3-2y) = 25+6\log(3-2y)-6\log(3-2y)$$
$$\Rightarrow y^2 = 25$$
$$\Rightarrow y = \pm 5$$

Thus, $y=5$ is the only value in the domain.
So the only solution to the equation is $y=5$.

(c) $(z-15)(z+1)^2 = (z+1)(z-15)\sqrt{(z-20)}$

Let $f(z) = (z-15)(z+1)^2$ & $g(z) = (z+1)(z-15)\sqrt{(z-20)}$.
Let $h(z) = \dfrac{1}{(z-15)(z+1)}$ be the quantity we want to multiply both sides by.
The domain of $f(z)$ is $\{z : z \in \mathbb{R}\}$

multiply both sides by.
The domain of $f(z)$ is $\{z : z \in \mathbb{R}\}$
The domain of $g(z)$ is $\{z : z \geq 20\}$ (Inside sqrt ≥ 0)
The domain of $h(z)$ is $\{z : z \neq 15 \text{ or } z \neq -1\}$ (Denominator > 0)
The intersections of the domains of f, g, & h is $\{z : z \geq 20\}$.
Now, consider

$$f(z) \cdot h(z) = g(z) \cdot h(z)$$

$$(z-15)(z+1)^2 \cdot \frac{1}{(z-15)(z+1)} = (z+1)(z-15)\sqrt{z-20} \cdot \frac{1}{(z-15)(z+1)}$$

$$z+1 = \sqrt{z-20}$$
$$(z+1)^2 = z-20$$
$$z^2+2z+1 = z-20$$
$$z^2+z+21 = 0$$

Use quadratic formula

$$z = \frac{-1 \pm \sqrt{1^2-4(1)(21)}}{2} = \frac{-1 \pm \sqrt{-83}}{2} \in \mathbb{C}$$

$\therefore$ there is no real solution to this equation.

(d) $\frac{1+w}{1+w} = 0$

Notice that $\frac{1+w}{1+w} = 1$. So this is saying $1=0$, which is obviously false. $\therefore$ there is no solution to this equation.

3. In which step is the error? **STEP 6.**

Notice that the only values that x & y can have are both 1. Thus, $x-y = 1-1 = 0$. In step 6, it says to divide both sides by $x-y$, AKA divide both sides by 0, which is not defined.

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Homework #12
4/13/20

4.3.2 labcdef, 4

1. Solve each equation in the set of real numbers. Indicate the function(s) you have applied to each side of the solution

a) $e^{(t^2)} = e^{3t}$

Applying the function $\log_b(t)$ to both sides

$\log(e^{(t^2)}) = \log(e^{3t})$

$t^2 = 3t$

Subtract $3t$ from both sides

$t^2 - 3t = 0$

Factor

$t(t-3) = 0$

$t = 0 \quad t - 3 = 0$

$\boxed{t = 0} \quad \boxed{t = 3}$

b. $\log_5(m^2) = 4$

Apply $a^{\log_a x} = x$

$5^{\log_5(m^2)} = 5^4$

$m^2 = 625$

subtract 625 from both sides

$m^2 - 625 = 0$

Factor

$(m-25)(m+25) = 0$

$\boxed{m = 25} \quad \boxed{m = -25}$

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c. $\log_5(p^2) = \log_5 4$

Apply $a^{\log_a x} = x$ to both sides

$5^{\log_5(p^2)} = 5^{\log_5 4}$

$p^2 = 4$

Subtract 4 from both sides

$p^2 - 4 = 0$

Factor

$(p+2)(p-2) = 0$

$\boxed{p = -2}$ $\boxed{p = 2}$

d. $6^z = 3$

Apply $\log_b x$ (base = 10) to both sides

$\log(6^z) = \log 3$

Power Property of Logarithms

$\log(6^z) = \log 3$

$z \log 6 = \log 3$

Divide both sides by $\log 6$

$$\boxed{z = \frac{\log 3}{\log 6}}$$

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e. $n^{12} = 12n$

Divide both sides by n

$$\frac{n^{12}}{n} = \frac{12n}{n}$$

$$n^{12-1} = 12$$

$$n^{11} = 12$$

Taking the 11th root of each side

$$(n^{11})^{\frac{1}{11}} = 12^{\frac{1}{11}}$$

$$\boxed{n = 12^{1/11}}$$

4. $\log_x 3 = \log_{2x} 6$

Change of Base Theorem → $\log_b x = \frac{\log_k x}{\log_k b}$

$$\frac{\log 3}{\log x} = \frac{\log 6}{\log 2x} \Rightarrow \frac{\log 3}{\log x} = \frac{\log(2\cdot 3)}{\log(2\cdot x)}$$

Product Property of Logarithms

$$\frac{\log 3}{\log x} = \frac{\log 2 + \log 3}{\log 2 + \log x}$$

$$\log 3(\log 2 + \log x) = \log x(\log 2 + \log 3)$$

$$\log 3 \cdot \log 2 + \log 3 \log x = \log 2 \cdot \log x + \log 3 \log x$$

$$\log 2 \log 3 = \log 2 \log x$$

$$\boxed{x = 3}$$

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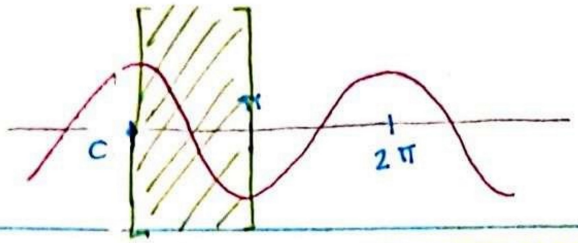


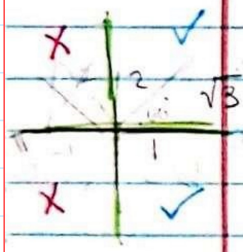
4. Solve each equation over the interval $[0, 2\pi)$. Give solutions to the nearest thousandth.

a) $\cos v = 0.5$

$\cos v = \frac{1}{2}$

- We would like to apply the inverse of the cosine function to both sides, but on the interval $[0, 2\pi)$ there is no inverse function.
- Restrict the interval to one on which cosine has an inverse.

Scratchwork:



Find values b/w 0 and π s.t. that $\cos v = \frac{1}{2}$ a

$$\cos v = \frac{1}{2}$$

$$\cos^{-1}(\cos v) = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{v = \frac{\pi}{3}}$$

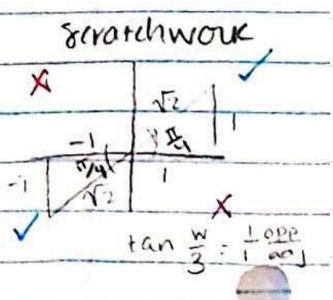
$$\boxed{v = \frac{5\pi}{3}}$$

b) $\tan \frac{w}{3} = 1$

- Restrict the interval to one on which tangent has an inverse
- Find values b/w $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ s.t. that

$$\tan \frac{w}{3} = 1$$

Scratchwork



NOTE:

- Then do the same for $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ to cover all of $[0, 2\pi)$

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the angles where $\tan \frac{w}{3} = 1$ are $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

We must solve for w,

$$\tan \frac{w}{3} = 1$$

To solve for w we must multiply by 3
Therefore multiply $\frac{\pi}{4}$ & $\frac{5\pi}{4}$ by 3

Giving us $\boxed{\frac{3\pi}{4}}$ & ~~$\frac{15\pi}{4}$~~

However $\frac{15\pi}{4}$ is not in the interval $[0, 2\pi)$, therefore not a solution.

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