SOLVING INEQUALITIES

4.3.3

WHY DO WE NEED INEQUALITIES?

Maria and Violet need to buy toilet paper and hand sanitizer. Walmart sells each roll of toilet paper for $1.65 and a bottle of hand sanitizer for $3. Albertsons sells each roll of toilet paper for $1.78 and a bottle of hand sanitizer for $2.80.

WHY DO WE NEED INEQUALITIES?

Maria and Violet need to buy toilet paper and hand sanitizer. Walmart sells each roll of toilet paper for $1.65 and a bottle of hand sanitizer for $3. Albertsons sells each roll of toilet paper for $1.78 and a bottle of hand sanitizer for $2.80.

Inequalities arise when we wish to know when one quantity is larger or smaller than another in terms of time, cost, rate, or any other attribute that a variable might measure.

WHAT IS AN INEQUALITY?

An inequality compares two values, showing if one is < (less than), > (greater than), ≤ (less than or equal to), ≥ greater than or equal to), or simply ≠ (not equal) to another value.

In other words, for any numbers a and b, either $a = b$ or $a \neq b$. If $a \neq b$ then there are two posibilities:

1. $a < b$
2. $a > b$

WHAT IS AN INEQUALITY?

An inequality compares two values, showing if one is < (less than), > (greater than), ≤ (less than or equal to), ≥ greater than or equal to), or simply ≠ (not equal) to another value.

In other words, for any numbers a and b, either $a = b$ or $a \neq b$. If $a \neq b$ then there are two posibilities:

1. $a < b$
2. $a > b$

One rule to remember!! If you multiply or divide by a negative number, you need to flip the inequality.

WHAT IS AN INEQUALITY?

An inequality compares two values, showing if one is < (less than), > (greater than), ≤ (less than or equal to), ≥ greater than or equal to), or simply ≠ (not equal) to another value.

In other words, for any numbers a and b, either $a = b$ or $a \neq b$. If $a \neq b$ then there are two possibilities:

1. $a < b$
2. $a > b$

One rule to remember!! If you multiply or divide by a negative number, you need to flip the inequality.

$Ex\ 1:\quad x + 5 \geq 8$

$$x \geq 8 - 5$$

$$x \geq 3$$

WHAT IS AN INEQUALITY?

An inequality compares two values, showing if one is < (less than), > (greater than), ≤ (less than or equal to), ≥ greater than or equal to), or simply ≠ (not equal) to another value.

In other words, for any numbers a and b, either $a = b$ or $a \neq b$. If $a \neq b$ then there are two possibilities:

1. $a < b$
2. $a > b$

One rule to remember!! If you multiply or divide by a negative number, you need to flip the inequality.

$Ex\ 1$: $x + 5 \geq 8$

$x \geq 8 - 5$

$x \geq 3$

$Ex\ 2$: $-x + 5 \geq 8$

$-x \geq 8 - 5$

$x \leq -3$

SETTING UP INEQUALITIES

Maria and Violet need to buy toilet paper and hand sanitizer. Walmart sells each roll of toilet paper for $1.65 and a bottle of hand sanitizer for $3. Albertsons sells each roll of toilet paper for $1.78 and a bottle of hand sanitizer for $2.80.

SETTING UP INEQUALITIES

Maria and Violet need to buy toilet paper and hand sanitizer. Walmart sells each roll of toilet paper for \$1.65 and a bottle of hand sanitizer for \$3. Albertsons sells each roll of toilet paper for \$1.78 and a bottle of hand sanitizer for \$2.80.

If we let W(t) be the cost of t toilet paper rolls with Walmart and if we let A(t) be the cost of t toilet paper rolls with Albertsons, which choice reflects the correct equations?

SETTING UP INEQUALITIES

Walmart : 1 toilet paper roll 1.65, 1 bottle hand sanitizer $3.00

Albertsons: 1 toilet paper roll $1.78 and a bottle of hand sanitizer for $2.80

SETTING UP INEQUALITIES

Walmart : 1 toilet paper roll 1.65, 1 bottle hand sanitizer $3.00

Albertsons: 1 toilet paper roll $1.78 and a bottle of hand sanitizer for $2.80

1. Walmart : $W(t) = 3t + 1.65$, Albertsons : $A(t) = 2.80t + 1.78$
2. Walmart : $W(t) = 3 + 1.65t$, Albertsons : $A(t) = 2.80 + 1.78t$

SETTING UP AN INEQUALITY

Set up an inequality to be solved to determine the amount where Walmart is preferred over Albertsons.

SETTING UP AN INEQUALITY

Set up an inequality to be solved to determine the amount where Walmart is preferred over Albertsons.

1. Walmart < Albertsons
2. Albertsons < Walmart
3. Walmart ≠ Albertsons
4. I have no idea

LET'S PUT IT ALL TOGETHER AND SOLVE IT

What do we know?

- Equations: $W(t) = 3 + 1.65t$, $A(t) = 2.80 + 1.75t$
- Inequality: Walmart < Albertsons

LET'S PUT IT ALL TOGETHER AND SOLVE IT

What do we know?

- Equations: $W(t) = 3 + 1.65t$, $A(t) = 2.80 + 1.75t$
- Inequality: Walmart < Albertsons

Our Inequality is

$$3 + 1.65t < 2.80 + 1.75t$$

SOLVING AN INEQUALITY

$$3 + 1.65t < 2.80 + 1.75t$$

t ?

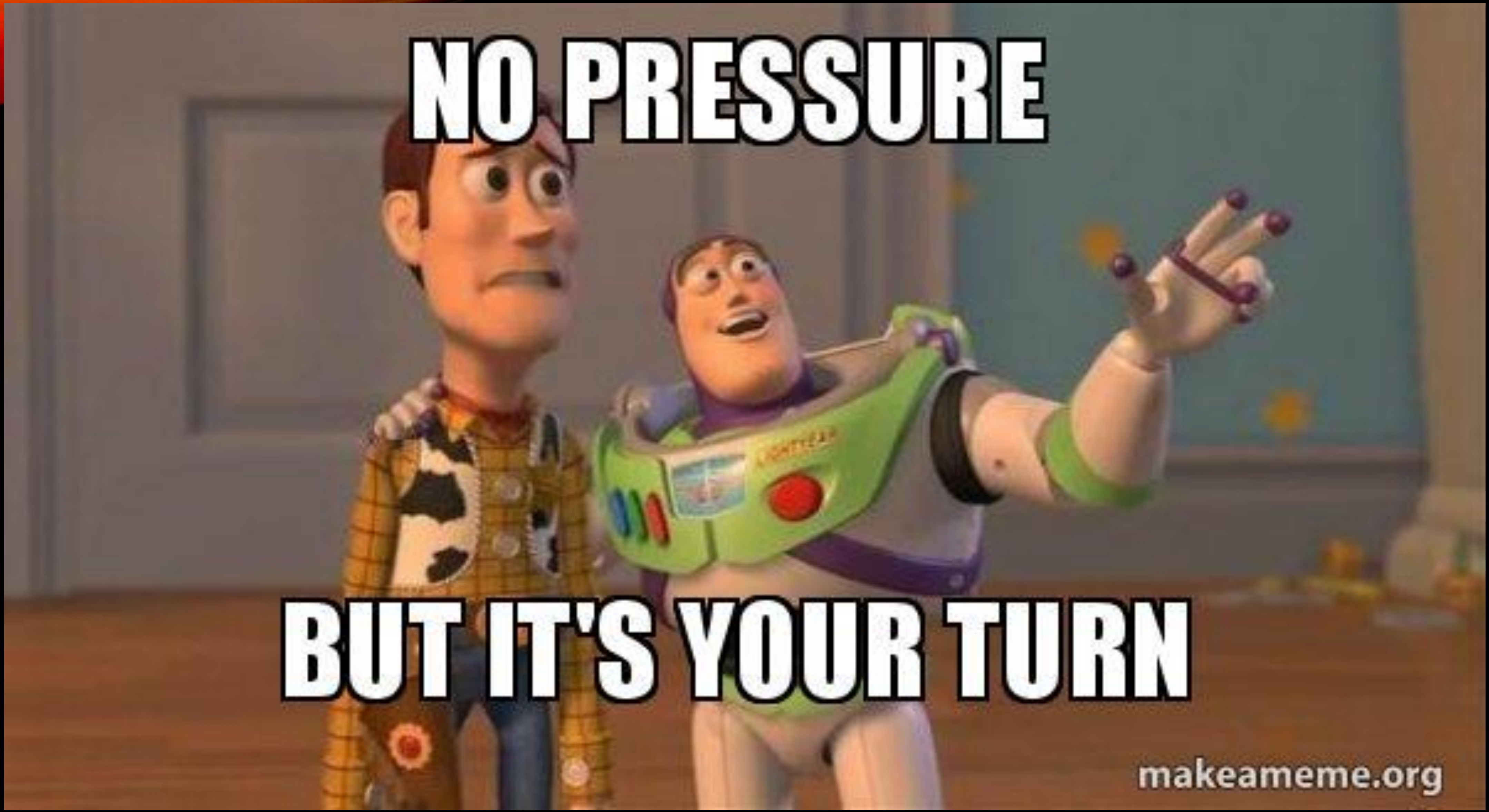
SOLVING AN INEQUALITY

The solution to our inequality was $t > 2$, but what does that mean?

SOLVING AN INEQUALITY

The solution to our inequality was $t > 2$, but what does that mean?

1. If more than 2 toilet papers are bought, then Walmart would cost less than Albertsons.
2. If more than 2 toilet papers are bought, then Albertsons would cost less than Walmart.


NO PRESSURE
BUT IT'S YOUR TURN
makeameme.org

Company A

$15 per month

21 cents per minutes

Company B

$20 per month

18 center per minute

Steps to solving an inequality

Step 1: Set up equations

Step 2: Set up Inequality

Step 3: Solve for your variable

Step 4: Interpret your solution

THEOREM 4.11 - $f(x) < g(x)$

For any continuous real functions f and g, with domain D:

a. If h is strictly increasing on the intersection of $f(D)$ and $g(D)$, then

$$f(x) < g(x) \Leftrightarrow h\big(f(x)\big) < h\big(g(x)\big).$$

b. If h is strictly decreasing on the intersection of $f(D)$ and $g(D)$, then

$$f(x) > g(x) \Leftrightarrow h\big(f(x)\big) > h\big(g(x)\big).$$

EXAMPLE OF THEOREM 4.11

Consider the inequality $f(x) > g(x)$, where the functions f and g and with $f(x) = a^x$ and $g(x) = b^x$. Let $h(x) = \log x$. Now h is increasing, therefore,

$$f(x) > g(x) \Leftrightarrow h(f(x)) > h(g(x))$$

EXAMPLE OF THEOREM 4.11

Consider the inequality $f(x) > g(x)$, where the functions f and g and with $f(x) = a^x$ and $g(x) = b^x$. Let $h(x) = \log x$. Now h is increasing, therefore,

$$f(x) > g(x) \Leftrightarrow h(f(x)) > h(g(x))$$

If we substitute our values of $f(x)$, $g(x)$, and $h(x)$, what do we end up with?

EXAMPLE OF THEOREM 4.11

Consider the inequality $f(x) > g(x)$, where the functions f and g and with $f(x) = a^x$ and $g(x) = b^x$. Let $h(x) = \log x$. Now h is increasing, therefore,

$$f(x) > g(x) \Leftrightarrow h(f(x)) > h(g(x))$$

We get

$$a^x > b^x \Leftrightarrow \log a^x > \log b^x$$

Use a power log on the right-hand side, and what do we get?

EXAMPLE OF THEOREM 4.11

Consider the inequality $f(x) > g(x)$, where the functions f and g and with $f(x) = a^x$ and $g(x) = b^x$. Let $h(x) = \log x$. Now h is increasing, therefore,

$$f(x) > g(x) \Leftrightarrow h(f(x)) > h(g(x))$$

We get

$$a^x > b^x \Leftrightarrow \log a^x > \log b^x$$

We get:

$$a^x > b^x \Leftrightarrow x \log a > x \log b$$

EXAMPLE OF THEOREM 4.11

$$a^x > b^x \Leftrightarrow x \log a > x \log b$$

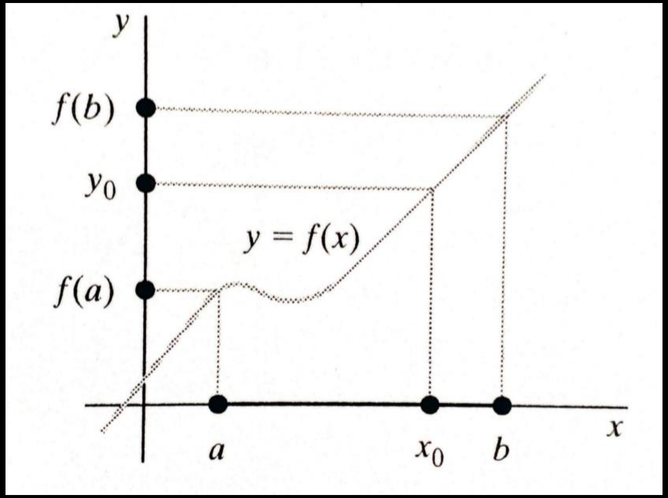
Remembering our rule, if $x > 0$, we can divide by x without changing the inequality. On the other hand, if $x < 0$, the division by x will flip the inequality.

Therefore, when $a^x > b^x$, then $a > b$ if x is positive, and $a < b$ if x is negative.

INTERMEDIATE VALUE THEOREM

Suppose f is a continuous function on the interval $[a, b]$.

Then for every real number between $f(a)$ and $f(b)$, there is at least one real number x_0 between a and b such that $f(x_0) = y_0$.



SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

Step 1: Find the zeros of f

$$f(x) = \frac{(x+1)(x-3)^2}{5x+6}$$

SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

Step 1: Find the zeros of f

$$f(x) = \frac{(x+1)(x-3)^2}{5x+6}$$

$x + 1 = 0$

$x = ?$

$(x-3)^2 = 0$

$x = ?$

$5x + 6 = 0$

$x = ?$

SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

Step 1: Find the zeros of f

$$f(x) = \frac{(x+1)(x-3)^2}{5x+6}$$

$x + 1 = 0$

$x = -1$

$(x-3)^2 = 0$

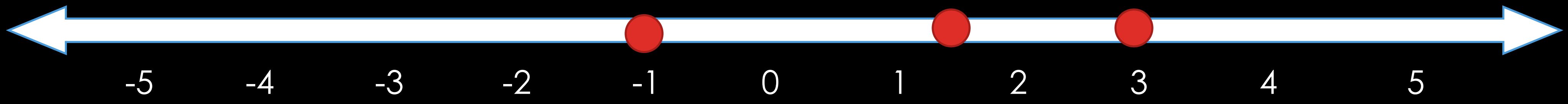
$x = 3$

$5x + 6 = 0$

$x = 1.2$

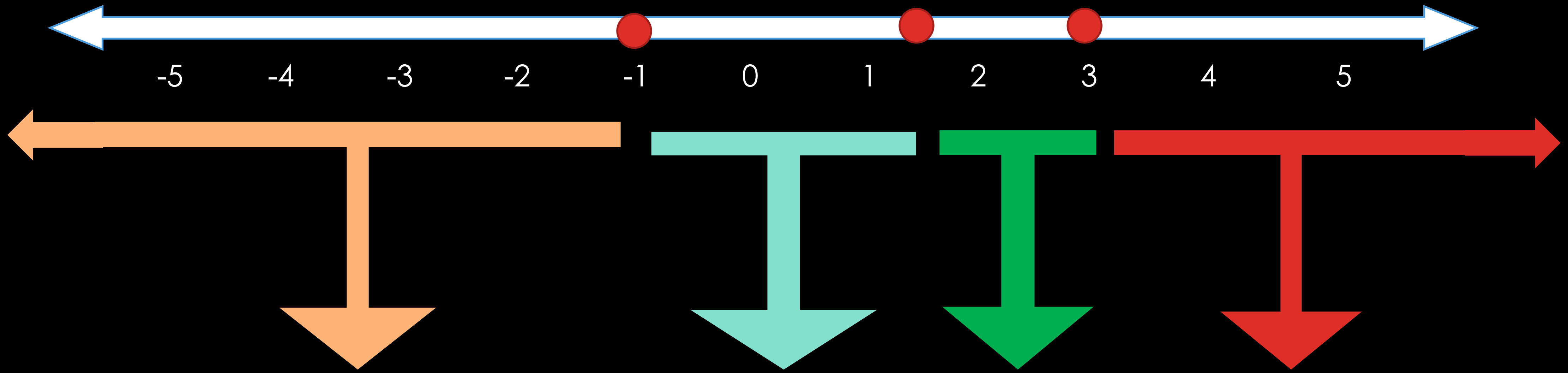
SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

Step 2: Find your intervals



SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

Step 2: Find your intervals



1st Interval $(-\infty, -1)$

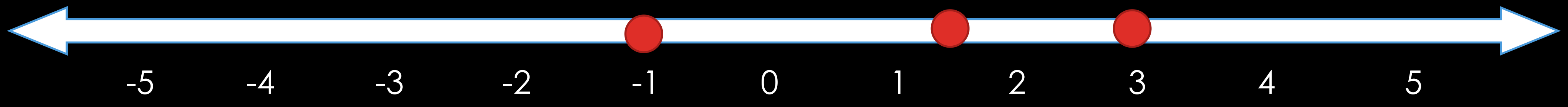
2nd Interval $(-1, 1.2)$

3rd Interval $(1.2, 3)$

4th Interval $(3, \infty)$

SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

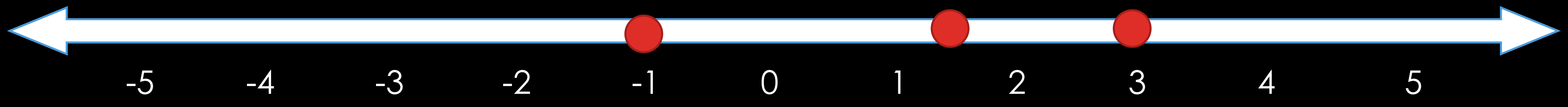
Step 3: Find a representative value of f for x in each interval



Interval $(-\infty, -1)$: We let $x = 2$ $f(-2) = \frac{-1(-5)^2}{-16} = \frac{25}{16} > 0$

SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

Step 3: Find a representative value of f for x in each interval



Interval $(-\infty, -1)$:

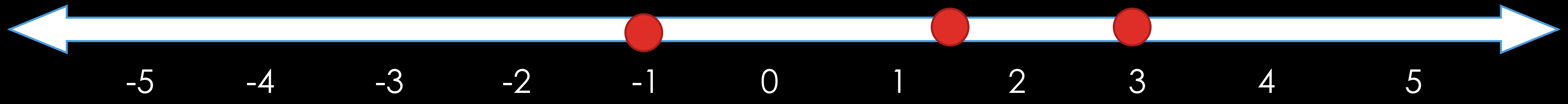
We let $x = 2$

$f(-2) = \frac{-1(-5)^2}{-16} = \frac{25}{16} > 0$

TRUE

SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

Step 3: Find a representative value of f for x in each interval



Interval $(-\infty, -1)$:	We let $x = 2$	$f(-2) = \frac{-1(-5)^2}{-16} = \frac{25}{16} > 0$	TRUE
Interval $(-1 \,,\; 1.2)$:	We let $x = 0$	$f(0) = \frac{1(-3)^2}{-6} = -1.5 > 0$	FALSE
Interval $(1.2 \,,\; 3)$:	We let $x = 2$	$f(2) = \frac{3(-1)^2}{4} = \frac{3}{4} > 0$	TRUE
Interval $(3 \,,\; \infty)$:	We let $x = 5$	$f(5) = \frac{6(2)^2}{19} = \frac{24}{19} > 0$	TRUE

SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

Step 3: Find a representative value of f for x in each interval

Interval $(-\infty, -1)$: We let $x = 2$ $f(-2) = \frac{-1(-5)^2}{-16} = \frac{25}{16} > 0$ **TRUE**

Interval $(-1\ ,\ 1.2)$: We let $x = 0$ $f(0) = \frac{1(-3)^2}{-6} = -1.5 > 0$ **FALSE**

Interval $(1.2\ ,\ 3)$: We let $x = 2$ $f(2) = \frac{3(-1)^2}{4} = \frac{3}{4} > 0$ **TRUE**

Interval $(3\ ,\ \infty)$: We let $x = 5$ $f(5) = \frac{6(2)^2}{19} = \frac{24}{19} > 0$ **TRUE**

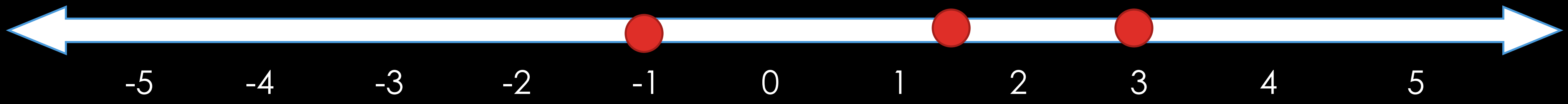
Thus, the solution set to $\frac{(x+1)(x-3)^2}{5x+6} > 0$ is $(-\infty, -1) \cup (1.2\ ,\ 3) \cup (3\ ,\ \infty)$

SOLVE $\frac{(x+1)(x-3)^2}{5x+6} > 0$

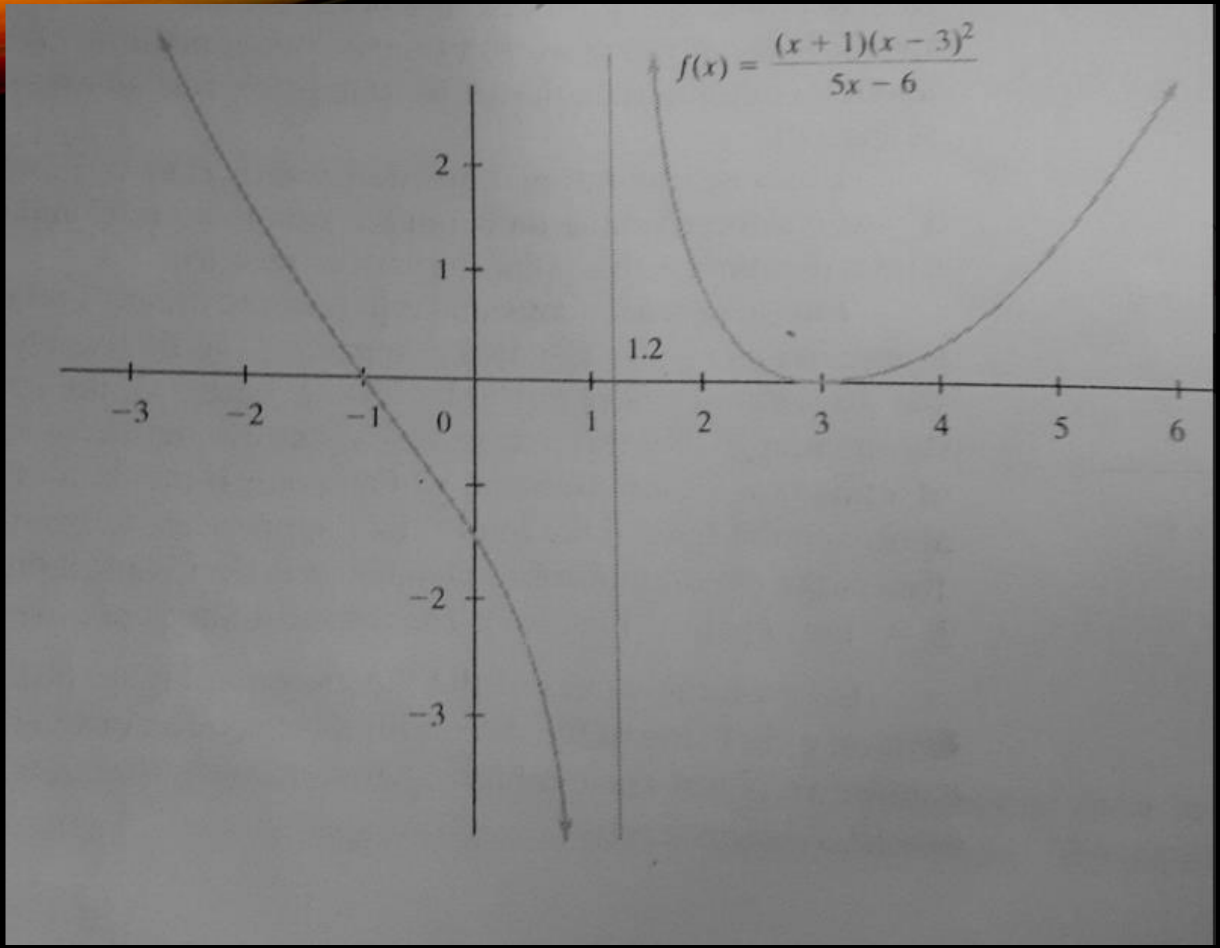
Intermediate Value Theorem:

Suppose f is a continuous function on the interval $[a, b]$.

Then for every real number between $f(a)$ and $f(b)$, there is at least one real number x_0 between a and b such that $f(x_0) = y_0$.



Thus, the solution set to $\frac{(x+1)(x-3)^2}{5x+6} > 0$ is $(-\infty, -1) \cup (1.2\ ,\ 3) \cup (3\ ,\ \infty)$



Thus, the solution set to $\frac{(x+1)(x-3)^2}{5x+6} > 0$ is $(-\infty, -1) \cup (1.2\,,\ 3) \cup (3\,,\ \infty)$

INTERMEDIATE VALUE THEOREM

SOLVE $\frac{x^2+10x+25}{x-5} > 6$

<u>Step 1: Find your zeros</u>

$$f(x) = [\left(\frac{x^2 + 10x + 25}{x - 5}\right) - 6]$$

<u>Step 2: Find your intervals</u>

-5 -4 -3 -2 -1 0 1 2 3 4 5

<u>Step 3: Find a representative of f for x in each interval</u>

Interval	Let $x = ?$	$f(\ ) = [\left(\frac{(\ )^2+10(\ )+25}{(\ )-5}\right) - 6]$