SIR Models for Infectious Diseases

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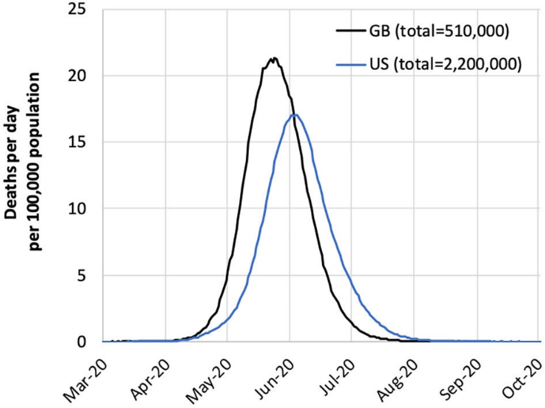
March 31, 2020

A grim forecast:



Source: Imperial College COVID-19 Response Team

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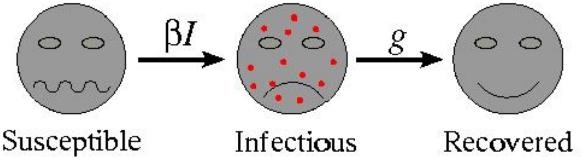
- $S(t)$ will be the **susceptible** population: those individuals who are not immune to the disease and can get sick.
- $I(t)$ is the **infectious** population: those individuals who are sick and can spread the disease to the susceptible population.
- $R(t)$ is the **recovered** population: those individuals who have had the disease and are now immune or who have died from the disease. (This also includes vaccinated individuals.)

Susceptibles will become infectious and then recover.

In the model the population is constant (in our case the population will be 100).



The SIR system of differential equations:

- $S'(t) = -\beta S(t) I(t)$
- $I'(t) = \beta S(t) I(t) - \gamma I(t)$
- $R(t) = 100 - S(t) - I(t)$

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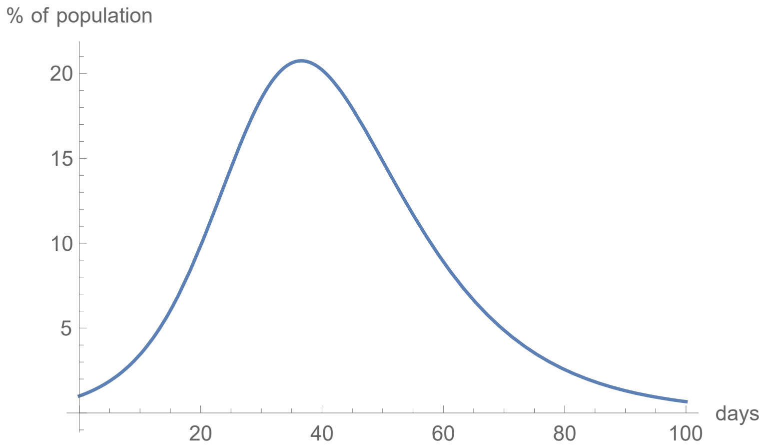
$1/\gamma$ is the **infectious period**, the time period an individual is infectious.

Graph of $I(t)$



$\beta = 0.0023 \quad \gamma = 1/10 \quad I(0) = 1$

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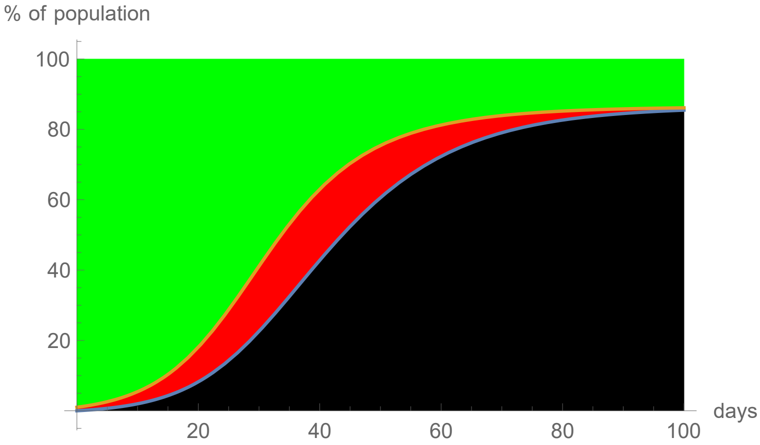


susceptibles infectious recovered

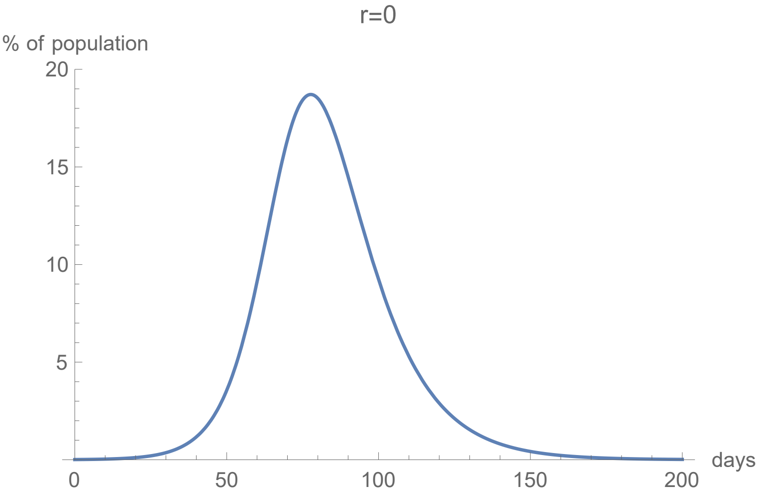
$\beta = 0.0023 \quad \gamma = 1/10 \quad I(0) = 1$

The SIR Model with social distancing:

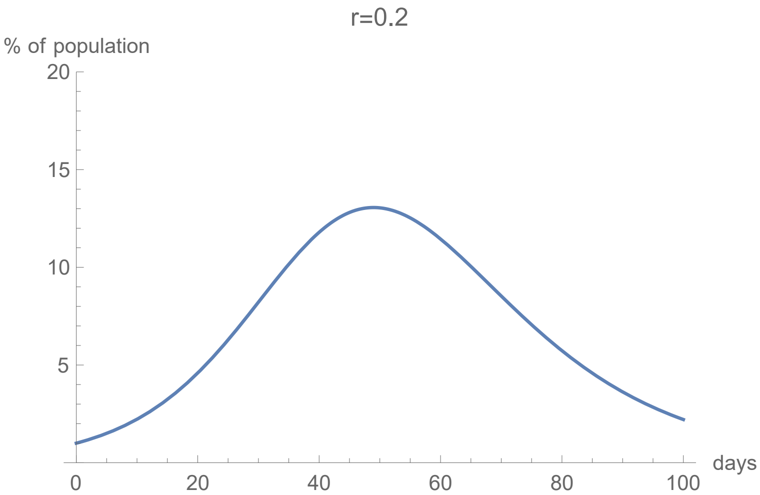
- $S'(t) = -\beta(1 - r)S(t)I(t)$
- $I'(t) = \beta(1 - r)S(t)I(t) - \gamma I(t)$
- $R(t) = 100 - S(t) - I(t)$

The parameter r describes the reduction of the contact rate (in %)

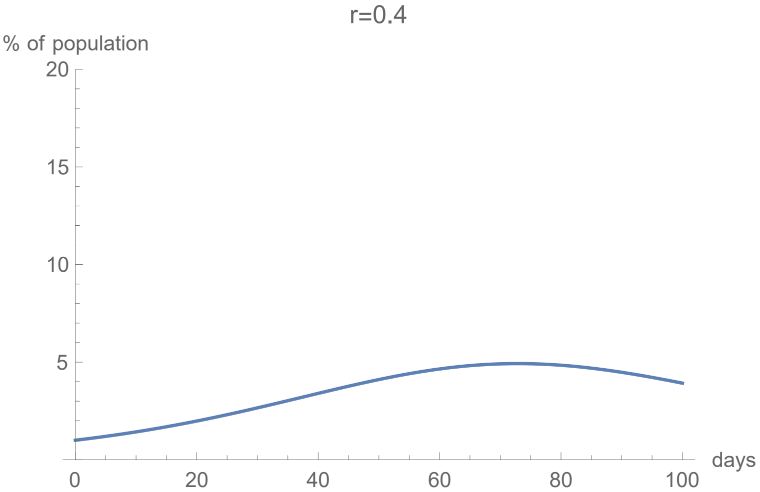
Graph of $I(t)$ with social distancing



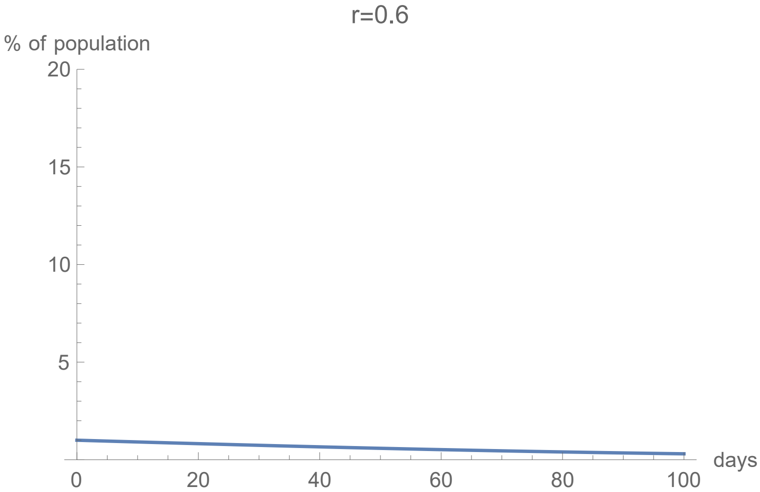
Graph of $I(t)$ with social distancing



Graph of $I(t)$ with social distancing



Graph of $I(t)$ with social distancing



References

- Matthew Keeling, The mathematics of diseases, Plus Magazine, retrieved 3/31/2020.
- See also other articles in *Plus Magazine* about the COVID-19 virus.