This is an open test: you are free to use any resources you want. **Prohibited** is any contact with classmates or other people during the exam. **Show all your work!**You need to email the test back to me by 16:25. PDF preferred, photos accepted if necessary. The test has five problems on five pages.

Problem 1 (20 points) 1. Let M be the set of real-valued 2×2 matrices with determinant -1:

 $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = -1 \right\}.$

Find matrices $A, B \in M$ such that the equation $A \cdot X = B$ does not have a solution $X \in M$.

Let A be any matrix in M. The equation $A \cdot X = A$ has a unique solution, manely the identity matrix. But $10l_2 \not\in M$ since det $10l_2 = +1$. More generally (M, \bullet) is not a group, fince it does not have a newboal element, f. i.

2. Let N be the set of real-valued 2×2 matrices with determinant 1. Does every equation $A \cdot X = B$, with $A, B \in N$, have a unique solution $X \in N$? Explain!

(N,.) is a group! Idz EN is the newbal elevent, and since det(AT) = (det A), N is closed under inversion: AEN => ATEN. Thus exerg equation A:X = B lus a unique solution in N.

Problem 2 (20 points)

Find the rule, the largest possible domain and the range for the function whose graph is the parabola passing through the points (-1, -5), (0, -4), and (1, -1).

Regeneral furctional rule for a fundratic function is PCX = ax2 + bx + C Usry the information given re totain 3 linear equetions (1) Q-6+C=-5 (2) [C=-4] (3) a+8+c=-1 (2)-(1) yield; 26=4=>18=21 Finally (3) yields. [a=-1-6-c=] So Pan= x2+2x-4 (= (x+1)2-5) the parabola is concave up, with its Vertex at x=-1 =, f(-1) = -5 Thy range P = [-5,00) of course, down? = IR

Problem 3 (20 points) 1. Find all real solutions in exact form of the equation

$$(x^2 + 4x + 4)^2 = -(x^2 + 5x + 6)^2$$

Explain carefully why your algebraic solving procedure is justified.

2. Find all real solutions in exact form of the equation

$$\ln(21 - x^2) = \ln(1 - x)$$

Explain carefully why your algebraic solving procedure is justified.

The domain of the equation is the interval (- ver 1) Cucks is a 1-1 function on its domain so

(=)
$$21-x^2 = 1-x$$

(=) $x^2-x-20=0$

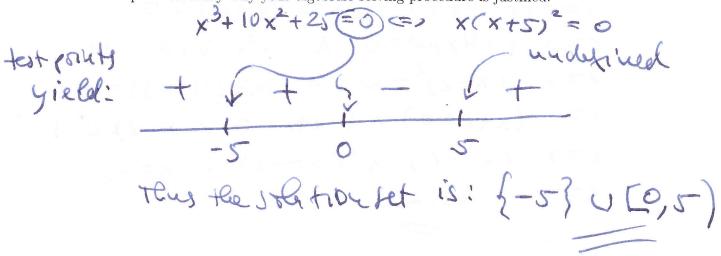
$$(=)$$
 $\chi^2 - \chi - 20 = 0$

$$(=)$$
 $(x-5)(x+4)=0$

Problem 4 (20 points) 1. Find all real numbers x that solve the inequality

$$\frac{x^3 + 10x^2 + 25x}{x - 5} \le 0.$$

Explain carefully why your algebraic solving procedure is justified.



2. Find all real numbers x that solve the inequality

$$\log_x 4 > \log_{2x} 16.$$

Explain carefully why your algebraic solving procedure is justified.

I will assume that x > 1. Then $f(y) = (2x)^{y}$ and $g(y) = x^{y}$ are shrittly increasing functions.

Log_x $+ x > \log_{2x} 16$ $= (2x)^{1/2} \times (2x)^{1$

Problem 5 (20 points) The set I of isometries of the complex plane forms a group with the binary operation of composition. Consider the following two elements in I:

$$g(z) = 3i + \overline{z}i,$$
 $h(z) = 5 - zi.$

1. Check that h is indeed an isometry.

2. Find the unique $f \in I$ such that $g \circ f = h$. Check that your f is an element in I.

(=)
$$3+f(z) = -5i \neq 2$$

(=) $f(z) = (-3-5i) \neq 2$