Math 4303 **Worksheet #3**

Let $\mathcal{K}$ denote the set of 2×2 matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, where $a, b \in \mathbb{R}$.

1. Show: If A and B are elements in $\mathcal{K}$, then $A + B \in \mathcal{K}$.
2. Show: If A and B are elements in $\mathcal{K}$, then $A \cdot B \in \mathcal{K}$.
3. Show that $A \cdot B = B \cdot A$ holds for all elements $A,\ B \in \mathcal{K}$.
4. Note that the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is an element of $\mathcal{K}$. Show: If the matrix $A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ in $\mathcal{K}$ satisfies $a^2 + b^2 \neq 0$, then A has a matrix inverse. What is A^{-1}? Does A^{-1} always lie in $\mathcal{K}$?
5. Show that we can identify $\mathcal{K}$ with $\mathbb{C}$, i.e., find a bijection $f : \mathcal{K} \to \mathbb{C}$ such that $f(A + B) = f(A) + f(B)$ and $f(A \cdot B) = f(A) \cdot f(B)$. (This is easier than it sounds.)
6. Under this identification, find the element I in $\mathcal{K}$ that corresponds to $i \in \mathbb{C}$ (not surprisingly, there are actually two possible choices). Compute $I \cdot I$.
7. Under this identification, what is the "conjugate" of an element in $\mathcal{K}$, what is the "modulus" of an element in $\mathcal{K}$?