Math 4303 **Worksheet #4** **Spring 2020**

[This is basically Problem 6 in Section 3.1.2 of our textbook.]

An airplane makes a round trip where the one-way distance is 1000 km. On the out-leg the plane faces a headwind of 50 km/h, while on the return there is a tailwind of 50 km/h. If the speed of the plane in still air is 400 km/h, what is the total time for the trip?

a. *A qualitative argument.* Before you solve the problem, think about it in a "qualitative" way: Sketch a rough graph of a function giving the total time of the trip in terms of wind speed as the wind speed varies from 0 to 400 km/h.

Compared with the total time for a round trip with **no** wind, do you think the time for the round trip **with** the wind is less, the same, or more?

b. *A numerical answer.* Answer the question of the problem. Does your answer support your response in **a.**?

c. *The motion functions.* Use an alternative approach by modeling the situation with the motion functions (distance vs. time) of the plane's outbound and return trip. Graph these functions.

Does this approach lead to the same answer as the one you found in **b.**?

d. *A general answer.* The numerical answer does not reveal much about the structure of the situation. Solve the problem again, this time expressing the total time in terms of general parameters for the total distance, the air speed of the plane and the wind speed.

There are many different equivalent symbolic expressions that will express the total time. Try to "coax" the expression you arrive at into a simple form.

e. *The general answer refined.* Express the total trip time with **no** wind (call it t_0) in terms of the given parameters. Use t_0 to get a more revealing expression for the total time of the trip **with** wind.

There is a connection of this problem to special relativity through a "Lorenz transformation." Look this up and explain what the connection is.

f. *The dimensionless factor.* A dimensionless factor $\dfrac{1}{1-r^2}$, where r is the ratio of the wind speed to the plane's speed, appears in the expression for the total time found in **e.** Analyze this factor as a function of r, and graph this function.