Name: _____

Math 2326 **Test 1** **Spring 2021**

Show your work to receive credit! Carefully read the instructions for each problem.

You may only use writing utensils and a calculator. You are not allowed to use any other material (textbook, notes, other books, CyberBoard, electronic resources, communication devices, etc.)

Print out the test, write in the space provided. (If you don't have a printer, use your own paper - copy figures as good as you can and clearly mark the problem you are working on.)

When you have finished, scan your work or take photos with your cell phone, assemble into **one PDF file**. Then email the PDF file back to me at hknaust@utep.edu by 3:00 p.m.

The test has 5 problems on 5 pages. **Good luck!**

Problem 1 (20 points) Find the solution to the following initial-value problem **in explicit form**. (This means write the solution as $y(t) = \ldots$)

$$y' + 2ty = 6t \qquad y(0) = 4$$

This is a linear (and separable) DEQ.

Integration factor $\quad p = 2t$

$$I(t) = e^{\int p(t)dt} = e^{\int 2t\,dt} = e^{t^2}$$

Multiply by $I(t)$:

$$\underbrace{y'e^{t^2} + 2te^{t^2}y}_{(ye^{t^2})'} = 6te^{t^2}$$

$$(ye^{t^2})' = 6te^{t^2}$$

Integrate: $\quad ye^{t^2} = 3e^{t^2} + C$

Note $(e^{t^2})' = 2te^{t^2}$ by chain rule

thus $y(t) = 3 + Ce^{-t^2}$

Solve the IVP: $\quad y(0) = 4$

so $\quad 4 = 3 + Ce^0 = 3 + C \Rightarrow C = 1$

$\longrightarrow \quad y(t) = 3 + e^{-t^2}$

Problem 2 (20 points) Solve the following initial value problem in **explicit** form.

$$\frac{dy}{dt} = \frac{1}{(y-1)(t+1)}, \quad y(0) = 2.$$

This is a separable DEQ:

$$(y-1)\,dy = \frac{dt}{t+1}$$

Integrate on both sides:

$$\frac{y^2}{2} - y = \ln(t+1) + C$$

don't need abs. value since for IVP, $t=0$ and thus $t+1>0$

This is a quadratic equation in y.

Using the quadratic formula we get

$$y = 1 \pm \sqrt{1 + 2\ln(t+1) + C}$$

Since $y(0) = 2 > 1$ in the IVP, we can discard the "$-$" answer:

$$y = 1 + \sqrt{1 + 2\ln(t+1) + C}$$

$y(0) = 2$ yields $C = 0$

$$\rightarrow \quad y = 1 + \sqrt{1 + 2\ln(t+1)}$$

Problem 3 (20 points) A glass tank initially contains 15 gallons of salt water containing 6 pounds of salt. Suppose salt water containing 1 pound of salt per gallon is pumped into the top of the tank at a rate of 2 gallons per minute, while a well-mixed solution leaves the bottom of the tank at the same rate of 2 gallons per minute.

1. Write an initial-value problem that models the amount of salt in the tank over time. Do not forget to properly introduce all variables you are going to use.

Let $S(t)$ the amount of salt in the tank at time t (in lbs)
Every minute 2 lbs of salt enters the tank;
since the salt concentration in the tank is $\frac{S(t)}{15}$ $\frac{lbs}{g}$,
and the outflow rate is $2\ \frac{g}{min}$,
$2\ \frac{S(t)}{15}$ lbs of salt will leave the tank per minute

$$\rightarrow \frac{dS}{dt} = 2 - 2\ \frac{S(t)}{15}$$

2. Solve the initial value problem in **explicit** form!

This is a linear DEQ. $I(t) = e^{\frac{2}{15}t}$

$$\frac{dS}{dt} + \frac{2}{15} S = 2$$

$$\underbrace{S' e^{\frac{2}{15}t} + \frac{2}{15} e^{\frac{2}{15}t} S}_{(Se^{\frac{2}{15}t})'} = 2e^{\frac{2}{15}t}$$

$$(Se^{\frac{2}{15}t})' = 2e^{\frac{2}{15}t}$$

Solving and finding C yields

$$S(t) = 15 - 9e^{-\frac{2}{15}t}$$

Note that $\lim_{t \to \infty} S(t) = 15$, as expected!

Problem 4 (20 points) Use *Euler's Method* with step size $\Delta t = 1$ to approximate $y(1), y(2), y(3)$ and $y(4)$ for the initial value problem

$$y' = y - 2t, \quad y(0) = 1.$$

$t_0 = 0 \quad y_0 = \underline{1}$

$f(t,y) = y - 2t$

$t_1 = 0 + 1 = 1 \quad y_1 = 1 + f(0,1) \cdot 1 = 1 + 1 = 2$

$t_2 = 1 + 1 = 2 \quad y_2 = 2 + f(1,2) \cdot 1 = 2 + 0 = 2$

$t_3 = 2 + 1 = 3 \quad y_3 = 2 + f(2,2) \cdot 1 = 2 - 2 = 0$

$t_4 = 3 + 1 = 4 \quad y_4 = 0 + f(3,0) \cdot 1 = 0 - 6 = -6$

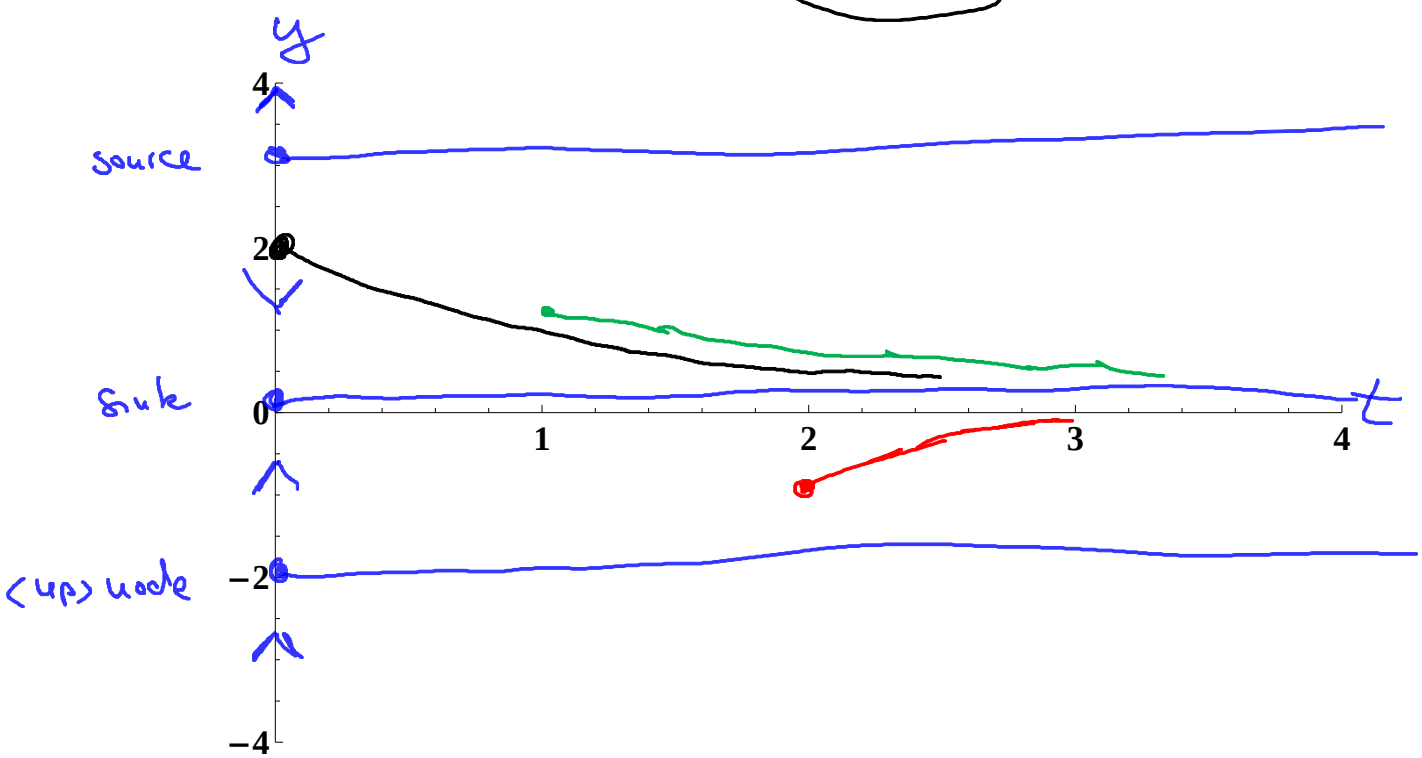
Problem 5 (20 points) Consider the autonomous differential equation

$$\frac{dy}{dt} = y(y-3)(y+2)^2.$$

1. Find the equilibrium points!

$\frac{dy}{dt} = 0 \Rightarrow y = 0$, $y = 3$ or $y = -2$

2. Insert the phaseline and the equilibrium solutions into the graph below. Then sketch the solution satisfying the initial condition $y(0) = 2$. (Do not forget to label the axes!)



3. Find $\lim_{t\to\infty} y(t)$ for the solution satisfying the initial condition $y(2) = -1$.

$\lim_{t\to\infty} y(t) = 0$

4. Find $\lim_{t\to\infty} y(t)$ for the solution satisfying the initial condition $y(1) = 1$.

$\lim_{t\to\infty} y(t) = 0$