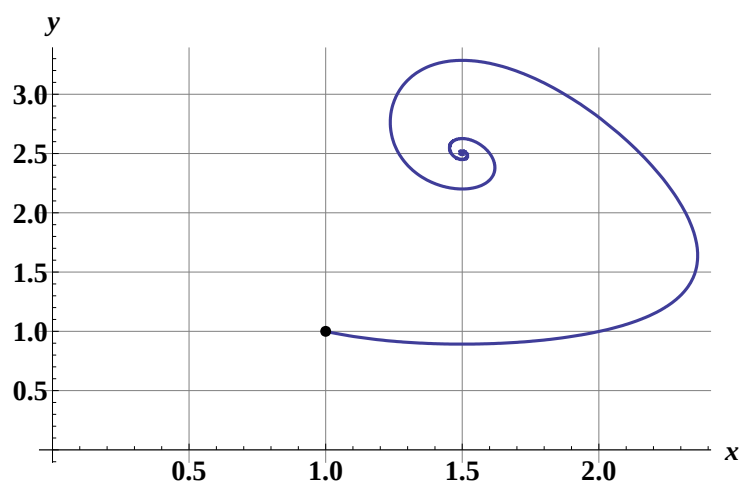
Name: Solutions

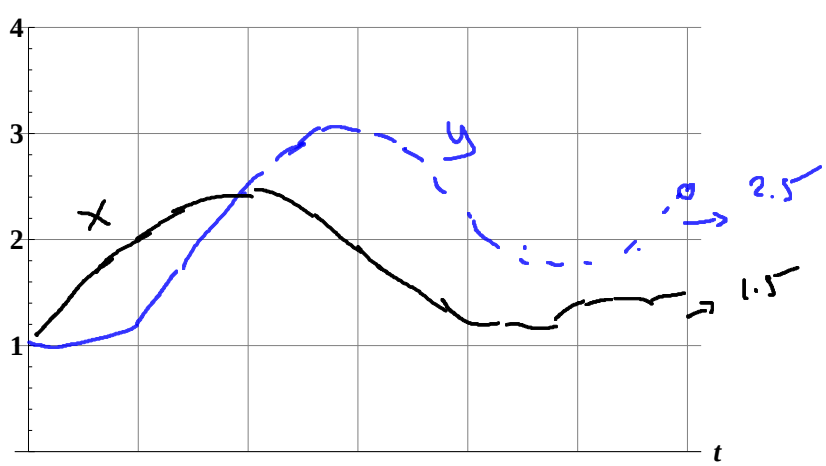
Math 2326 **Test 2** **Spring 2021**

Show your work to receive credit! Carefully read the instructions for each problem.
You may only use writing utensils and a calculator. You are not allowed to use any other material (textbook, notes, other books, CyberBoard, electronic resources, communication devices, etc.)
Print out the test, write in the space provided. (If you don't have a printer, use your own paper - copy figures as good as you can and clearly mark the problem you are working on.)
When you have finished, scan your work or take photos with your cell phone, assemble into **one PDF file**. Then email the PDF file back to me at hknaust@utep.edu by **3:00 p.m. sharp.**
The test has 5 problems on 6 pages. **Good luck!**

Problem 1 (20 points) Below is the phase portrait of the solution to a predator-prey system of autonomous differential equations satisfying the initial conditions $x(0) = y(0) = 1$.

In the coordinate system below that, give a sketch of the time series $x(t)$ and $y(t)$. Clearly indicate which of your two graphs is $x(t)$ and which one is $y(t)$!





Problem 2 (20 points) Consider the following system of differential equations describing a predator-prey model:

$$\begin{aligned} x' &= x(2-x) + 4xy \\ y' &= y - xy \end{aligned}$$

1. Which of the two variables represents the prey population?

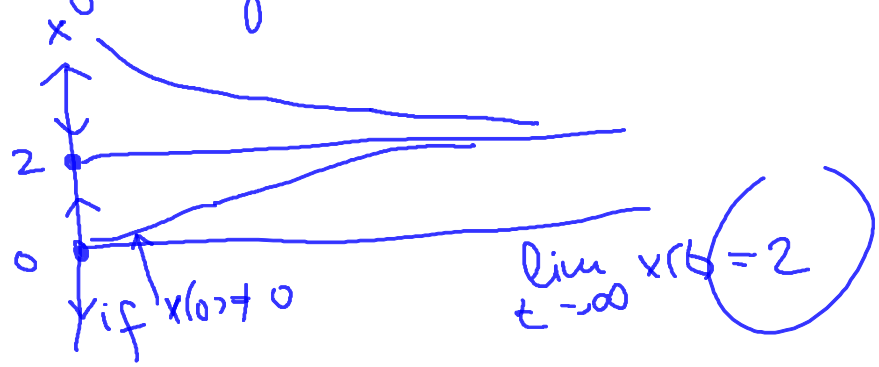
   y is the prey population b/c of the − sign in front of the xy term

2. Suppose $y(t_0) = 0$ for some time t_0. Explain why this implies that $y(t) = 0$ for all values of t.

   Then $y'(t_0) = 0$ as well, so y will not change i.e. $y(t) = 0$ for all t

3. Describe in words the fate of the predator population when the prey population becomes extinct. Then draw the phaseline for the predator population in this situation.

   The DEQ for x becomes $x' = x(2-x)$, logistic growth.



Problem 3 (20 points) Consider the following system of differential equations:

$$\begin{aligned} x' &= 2x \\ y' &= x^2 - 3y \end{aligned}$$

1. Find all solutions of this system.

The system is partially decoupled

$x' = 2x \Rightarrow x = Ae^{2t}$

$\Leftrightarrow y' = (Ae^{2t})^2 - 3y$

$\Leftrightarrow y' + 3y = A^2 e^{4t}$

$\Leftrightarrow y' e^{3t} + 3e^{3t} y = A^2 e^{7t}$

$\Leftrightarrow (ye^{3t})' = A^2 e^{7t}$

$\Leftrightarrow ye^{3t} = \frac{A^2}{7} e^{7t} + B$

$\Leftrightarrow y = \frac{A^2}{7} e^{4t} + Be^{-3t}$

2. Find the solution satisfying the initial conditions $x(0) = -7$ and $y(0) = 0$.

$x = -7e^{2t}$

$y = 7e^{4t} - 7e^{-3t}$

Problem 4 (20 points) Consider the system of linear differential equations $\mathbf{Y}' = A \cdot \mathbf{Y}$, where $A = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$.

1. Find the eigenvalues of the matrix A.

characteristic equ:

$$(2-\lambda)(0-\lambda) - 3 = 0$$

$$\Leftrightarrow \quad -2\lambda + \lambda^2 - 3 = 0$$

$$\Leftrightarrow \quad \lambda^2 - 2\lambda - 3 = 0$$

$$\Leftrightarrow \quad (\lambda+1)(\lambda-3) = 0$$

so $\lambda = -1$ or $\lambda = 3$

2. Find an eigenvector for each of the eigenvalues.

$\underline{\lambda = -1}$ $\quad A - \lambda Id = \begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix}$

eigenvector $\underline{v}$ satisfies

$(A - \lambda Id)\,\underline{v} = \underline{0}$

so we get the condition

$3v_1 + v_2 = 0$

Can take: $\underline{v} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\underline{\lambda = 3}$ $\quad A - \lambda Id = \begin{pmatrix} -1 & 1 \\ 3 & -3 \end{pmatrix}$

we get the condition $-v_1 + v_2 = 0$, so we can take $\underline{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

3. Write down the general solution of the system $\mathbf{Y}' = A \cdot \mathbf{Y}$.

$$Y = K_1 \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{-t} + K_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t}$$

Problem 5 (20 points) Consider the following family of autonomous differential equations parametrized by α:

$$\frac{dy}{dt} = y^2 + \alpha y$$

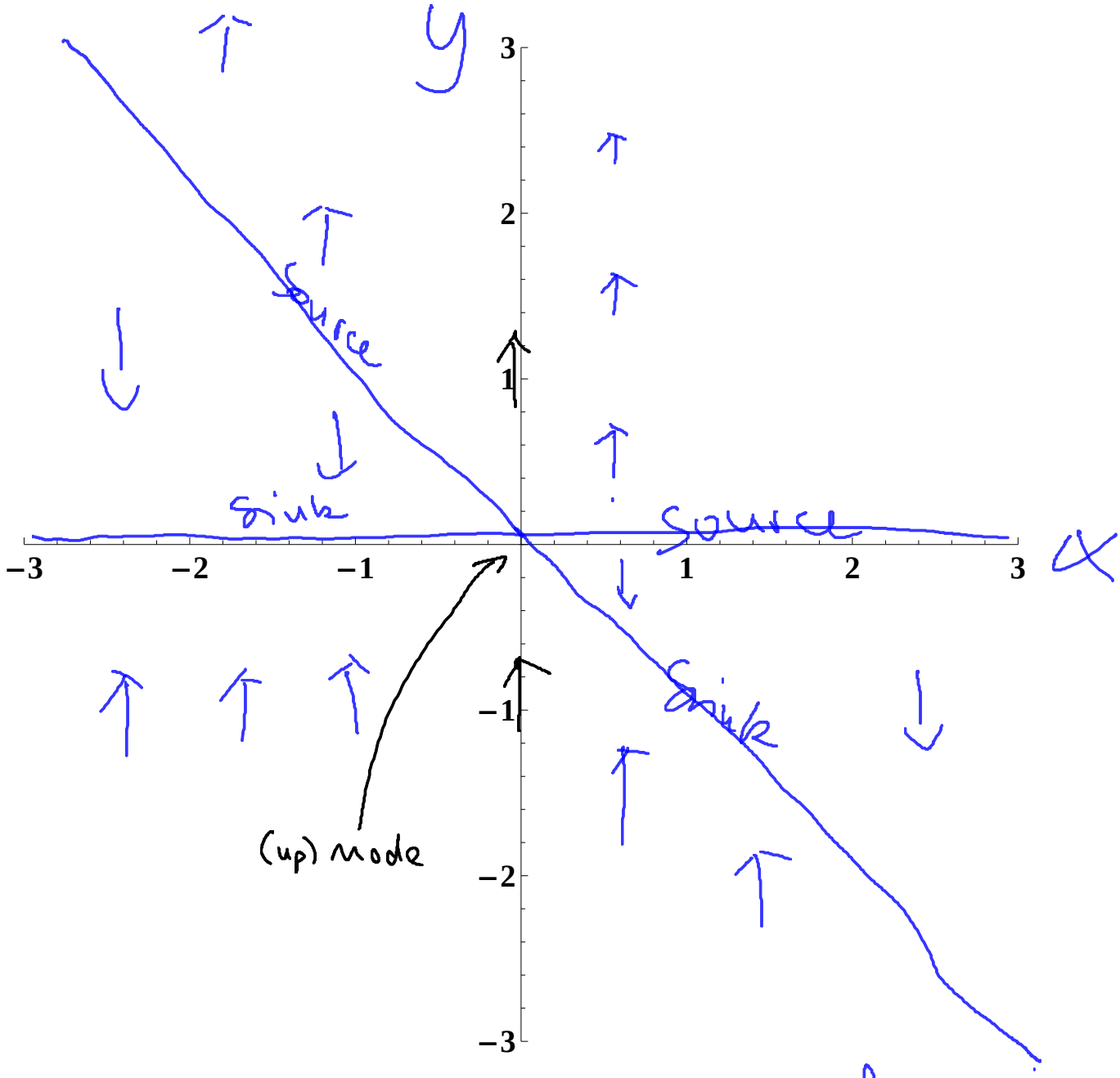
1. Sketch the bifurcation diagram of this family on the next page! Clearly indicate the locus of the equilibrium points and the phase line arrows in the various regions cut out by the locus of equilibrium points. As usual, do not forget to label the axes!

2. Find the bifurcation value(s) and describe how the qualitative behavior of the solutions changes at the bifurcation value(s).

locus of the equil. pts $\Big\}$ $y^2 + \alpha y = 0 \iff y(y+\alpha) = 0$

$\iff y = 0$ or $y = -\alpha$

(2 lines)



$\alpha = 0$ is the only bifurcation value

$$\frac{dy}{dt} = y^2 + \alpha y$$

	$\alpha < 0$	$\alpha = 0$	$\alpha > 0$
# equ. pts	2	1	2
	source sink	up-node	source sink