

**Predator-Prey Models and Hunting<sup>1</sup>**

Consider the classical predator-prey model

$$\begin{aligned}\frac{dx}{dt} &= Ax - Bxy \\ \frac{dy}{dt} &= -Cy + Dxy.\end{aligned}$$

Such a system typically has a periodic solution, i.e., there is some time constant  $T$  so that  $x(t + T) = x(t)$  and  $y(t + T) = y(t)$  for all  $t$ . Because of this periodic behavior, it is useful to consider the average populations  $\bar{x}$  and  $\bar{y}$ , defined by

$$\begin{aligned}\bar{x} &= \frac{1}{T} \int_0^T x(t) dt \\ \bar{y} &= \frac{1}{T} \int_0^T y(t) dt.\end{aligned}$$

1. Show that  $\bar{x} = C/D$  and that  $\bar{y} = A/B$ . *Hint:* Use the first equation above and the fact that  $x(0) = x(T)$  to show that

$$\int_0^T (A - By(t)) dt = \int_0^T \frac{x'(t)}{x(t)} dt = 0.$$

2. Assume that a percentage of both species is hunted; more precisely: Assume that the prey is hunted reducing its rate of change by a constant  $\epsilon$  times the prey population, while the predators are hunted reducing their rate of change by a constant  $\delta$  times the predator population. Write down a system of first-order differential equations describing this new predator-prey model with hunting.

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<sup>1</sup>This laboratory is based on a group project in “*Fundamentals of Differential Equations*” by R. Kent Nagle and Edward B. Saff.

3. Assume  $\epsilon < A$ . What effect does this model of hunting have on the average prey and predator populations?
4. Assume that only the predator is hunted. What effect does this model of hunting have on the average prey and predator populations?
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6. In a rural community, foxes prey mainly on rabbits, but occasionally include a chicken in their diet. The farmers decide to put a stop to the chicken killing by hunting the foxes. What do you predict will happen?