Math 2326 **Laboratory 3D**

Bifurcations in Linear Systems[1]

Consider the following system of first-order linear differential equations:

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} a & b \\ -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} x(t) \\ y(t) \end{pmatrix},$$

where a and b are real parameters.

1 For each value of a and b, classify the system's equilibrium points (as sinks, spirals, etc.). Draw a picture in the "ab"-plane, and indicate the regions corresponding to the various types (for instance: shade all (a, b) values for which the origin is a sink red, the values for which the origin is a spiral sink orange, and so forth). Be sure to include all the computations necessary to draw the picture.

As the values of a and b are changed and the point (a, b) moves from one region to another, the "equilibrium type" changes. Such a change is called a ***bifurcation****. A typical bifurcation occurs when a harmonic oscillator changes from being underdamped to being overdamped.*

2 Which of the bifurcations in your picture affect the long term behavior of the solutions?

3 What is happening at the boundary between regions?

[1]This laboratory is based on a student laboratory in *"Differential Equations"* by Blanchard, Devaney and Hall.