

Consider the non-linear differential system

$$\begin{aligned}x' &= x(-x - y + 70) \\ y' &= y(-2x - y + a),\end{aligned}$$

where a is a real parameter. Assume that a varies between the values -10 and 170.

- 1 Find all equilibrium points of the system. Their location and number will, of course, depend on a .
- 2 Classify the equilibrium points by linearization.
- 3 At what values of a does the number of equilibrium points change?
- 4 At what values of a does the “type” of an equilibrium point change (e.g., from sink to saddle)?
- 5 The system models a situation, where two animal species compete for limited resources. $x(t)$ and $y(t)$ denote the sizes of the two animal populations at time t .
Explain, why the differential system above might be a reasonable model for such a situation. What is the meaning of the parameter a ?
- 6 Can you make predictions about the long-term fate of the animal populations? How does the parameter a affect their fate?