Math 2326 # Hopf Bifurcation[1] **Laboratory 4C**

A certain oscillating chemical reaction (the chlorine dioxide-iodine-malonic acid reaction) can be modeled by a system of two first-order differential equations:

$$\begin{aligned} \frac{dx}{dt} &= 10 - x - \frac{4xy}{1+x^2} \\ \frac{dy}{dt} &= bx\left(1 - \frac{y}{1+x^2}\right). \end{aligned}$$

The variables x and y should be thought of as modeling concentrations of I^- and ClO_2^-, while b is a certain chemical constant.

1 Find the equilibrium point of the system above.
2 Linearize the system at the equilibrium point.
3 Let $b = 5$. What kind of equilibrium does the system have?
4 Let $b = 1$. What kind of equilibrium does the system have?
5 For which value of b does the system change its equilibrium type?
6 Let $b = 1$. Consider the rectangle

$$\begin{aligned} 0 \leq \ & x \ \leq 10 \\ 0 \leq \ & y \ \leq 101 \end{aligned}$$

Show (using pencil and paper calculations) that no solution will leave this rectangle! Proceed as follows: show that the direction field points to the right at the left border of the rectangle, that it points to the left at the right border, that it points up at the bottom border and that it points down at the top border. Explain carefully why this establishes that no solution can escape from the rectangle!
7 Let $b = 1$. Describe the long-term behavior of all solutions starting inside the rectangle. Explain how you arrive at your conclusions. **Hint:** Look at the solutions of the *van der Pol* equation in Figure 5.1 on page 458 of our textbook.

[1] This laboratory is based on Experiment 25 in the *"Interactive Differential Equations Workbook"* by West, Strogatz, McDill and Cantwell.

H. Knaust. April 28, 2021.