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Topic: Induction
Think about teaching children the counting numbers.
Meaning: 1,2,3,4,5,6 etc.…

What are some definitions that we have teach kids when they are first learning the counting numbers?

Dedekind (circa 1890s):
Axioms for Natural numbers:

There is a set of $N \in 1$
that is together nth a successor function

(D1) S: N -> N {1} (is onto function)
(D2) S: N -> N (one to one function)
(D3) If M is the subset that satisfies 1 element M
Wherever there is an element in M then S(M) is an element in M
M=N

For every number that we are counting there is always a number that will succeed the previous number.

Satisfying 1 is not a successor all other elements in N are successors.

Two distinct elements in N have distinct successors.

D3 is equivalent to D3' let P(m) be a statement the free variable M draws from N

IF P satisfies, then P is true then M is the 1 P(S(m)) is true for all elements in N
Then P(m) is true for all m elements in N.

{ Recursive definition: How to add
1) Add "1"0
   K + 1 = S(K)
2) If we know that k + n is

Then $k + S(n) = S(k+n)$

}

(Peamo)

Dedekind: How to use D1 and D2 to show that addition is now defined for all m?

Start with 1.

Next is 2.

So on and so forth.

Logicism:

G. Frege: what is logic?

G. Cantor: what are sets?

R. Dedekind: What are numbers?

Crash: B. Russel

Problem 3.3.

If *n* is a natural number, then the sum of the first *n* even numbers is $n^2 + n$.

I know that $S = \{2 + 4 + 6 + \ldots + 2n\} = n^2 + n$

I want $S = \{ 2+ 4 + 6 + \ldots + 2n + (2n+2)\} = (n+1)^2 + (n+1)$

$= 2 (1 + 2 + 3 + \ldots + n)$

$= \frac{2n(n+1)}{2}$

$= n (n+1)$

$= n^2 + n$

Thus, the sum of the first even natural numbers is $n^2 + n$

Problem 3.4

Let S = {1 + 3 + 5 + … + 2n-1}

We know that the first n natural numbers = $\frac{n(n+1)}{2}$

$= 2*\frac{n(n+1)}{2} - n$

$= n(n+1) - n$

$= n^2 + n - n$

$= n^2$

Thus, the sum of the first odd natural numbers is n^2