MATH 3325
Fall 2022

September 27, 2022

4.20: If $A \subseteq B$ & $B \subseteq C$, then $A \subseteq C$

Suppose $x \in A$. Since $A \subseteq B$; $x \in B$. Again, since $B \subseteq C$, $x \in C$.

4.21: $A \cap U = A$

$A \subseteq U$, therefore if $x \in A$, then $x \subseteq U$, which means that all $x \in A$ are common $A \cap U = A$

If $x \in A \cap B$, then because of Definition 4.7, $x \in A$, and $x \in U$

Def 4.7 $A \cap B$ consists of all elements that are in A & B

$$A \cap B = \{x \in U \mid x \in A \wedge x \in B\}$$

$$A \cup B = \{x \in U \mid x \in A \vee x \in B\}$$

$A = B \leftrightarrow A \subseteq B$ and $B \subseteq A$

4.21 $A \cup \emptyset = A$

case 1: $A \cup \emptyset \subseteq A$

Let $x \in A \cup \emptyset$

$\Leftrightarrow$ $x \in A$ or $x \in \emptyset$, by def of $\emptyset$ & by def of $\cup$

$\Rightarrow$ $x \in A$

case 2: $A \subseteq A \cup \emptyset$

let $x \in A$

$x \in A \cup \emptyset$, by def $\cup$

4.22 $A \subseteq A$

let $x \in A$; then $x \in A$

$\emptyset \subseteq A$

if $x \in \emptyset$, then $x \in A$

$x \in \emptyset$ is always false; thus the implication is true

4.23 $A \cup A^c = U$

Case 1: $A \cup A^c = U$

let $x \in A$ & $x \in A^c$

by definition of union

By def. of complement

$A \cup A^c \subseteq U$

Case 2: $U \subseteq A \cup A^c$

let $x \in U$

$x \in U \iff x \in A \; x \in A^c$

by def of union

$x \in A$ or $x \in A^c$

$x \in A \cup A^c$

4.25 $(A \cup B) \cup C = A \cup (B \cup C)$

"$\subseteq$"

if let $x \in (A \cup B) \cup C$

then ($x \in A$ or $x \in B$) or $x \in C$

4.26. Follow Problem 1.13

4.27. Follow Problem 1.15

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Problem 4.28 Prove the following statements

1. $(A \cap B)^C = A^C \cup B^C$
2. $(A \cup B)^C = A^C \cap B^C$

De Morgan's Laws $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

Let $x \in (A \cap B)^C$

so $x \notin A \wedge B$

$\Leftrightarrow \neg(\overbrace{x \in A}^{P} \quad \wedge \quad \overbrace{x \in B}^{Q})$

$\Leftrightarrow \neg(x \in A) \quad \vee \neg(x \in B)$

$\Leftrightarrow x \notin A \quad \vee \quad x \notin B$

$\Leftrightarrow x \in A^C \vee \quad x \in B^C$

$\Leftrightarrow x \in A^C \cup B^C$