Class notes oct. 4

3.16

let $a_1 = 1$, $a_2 = 2$, $a_3 = 3$ and for each natural number greater than 3 define: $a_n = a_{n-1} + a_{n-2} + a_{n-3}$

$P(n)$ = prove that $a_n < 2^n \ \forall n \in \mathbb{N}$

Base case

$a_1 = 1 < 2^1$

$a_2 = 2 < 2^3$

$a_3 = 3 < 2^3$

Induction step

If $m \in \mathbb{N}$ then
$P(j \leq m) \Rightarrow P(M+1)$

$j \in \mathbb{N}$

Know: $a_1 < 2^1$ Want: $a_{m+1} < 2^{m+1}$

Since $m \geq 3$ $m+1 > 3$

$$a_{m+1} = a_m + a_{m-1} + a_{m-2} < 2^m + 2^{m-1} + 2^{m-2}$$
$$< 2^m + 2^{m-1} + 2^{m-1}$$
$$\leq 2^{m+1}$$

Therefore $P(n)$ is true $\forall n \in \mathbb{N}$

4.30

$A \setminus (A \setminus B) = A \cap B$

PF: let $x \in A \setminus (A \setminus B)$

$= A \setminus (A \cap B^c)$

$= A \cap (A \cap B^c)^c$

$= A \cap (A^c \cup B)$

$= (A \cap A^c) \cup (A \cap B)$

$= (A \cap B)$

let $x \in A \setminus (A \setminus B)$

$\Leftrightarrow x \in A \setminus (A \cap B^c)$

$\vdots$ and so on

Note when using $\Leftrightarrow$ you need statements

4.31

$A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$

let $x \in A \setminus (B \setminus C)$

$\Leftrightarrow x \in A$ and $x \notin (B \setminus C)$

$\Leftrightarrow x \in A$ and $x \notin B$ or $x \in C$

$\Leftrightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \in C)$

$\Leftrightarrow (x \in A \setminus B) \cup (x \in A \cap C)$

$\Leftrightarrow x \in (A \setminus B) \cup (A \cap C)$

4.36 $A = \{\square, \triangle, \bigcirc\}$

$B = \{\bullet, \bar{\bullet}, \underline{\bullet}\}$

A x A
A x B
B x A
B x B

$A \times B = \{(\square, \bullet), (\square, \bar{\bullet}), (\square, \underline{\bullet}) \ldots (\bigcirc, \bullet)\}$

$B \times A = \{(\bullet, \square), (\bullet, \triangle), (\bullet, \bigcirc), \ldots (\underline{\bullet}, \bigcirc)$

$A \times A = \{(\square, \square), (\square, \triangle), (\square, \bigcirc), (\triangle, \square), \ldots, (\bigcirc, \bigcirc)\}$

$A \times B = \{(\bullet, \bullet), (\bullet, \bar{\bullet}), (\bullet, \underline{\bullet}), \ldots, (\underline{\bullet}, \underline{\bullet})\}$

Why aren't they the same

$A \times B \neq B \times A$ because they are ordered pairs $(\square, \bullet) \neq (\bullet, \square)$

They are not equal

$U = (A \cup B) \times (A \cup B)$

$B \times A = \{(b, a) \mid (a, b) \in A \times B\}$

$A \times B = \{(a, b) \mid (a, b) \in A \times B)$

4.41

$A \times (B \times C) \stackrel{?}{=} (A \times B) \times C$

(a,b,c) $((a,b),c)$

ordered pair

$= \{\{(a,b)\},\{(a,b),c\}\}$

$= \{\{a\},\{a,(b,c)\}\}$

$= \{\{a\},\{a,\{\{b\},\{b,c\}\}\}\}$

$= \{\{\{a\},\{a,b\}\}\{\{\{a\},\{a,b\}\},c\}$

They are not the same

NOTE: $A \times B \times C = \{(a,b,c) \mid a \in A, b \in B, c \in C\}$

Remember

$(x,y) = \{\{x\},\{x,y\}\}$

$(y,x) = \{\{y\},\{x,y\}\}$

they are not the same / different set pairs

$\{x\}, \{x,y\}$

containing the first element which comes 1st.