October 11, 202

PoM Notes

3.11: If n is a natural number and $n \geq 4$, then $3^n > 2n^2+3n$

① Base case: $n=4$ $\quad 3^{(4)} > 2(4)^2+3(4) \iff 81 > 44$ ✓

② Deduction step: (k) $\quad 3^k > 2(k)^2+3(k)$, $k \in \mathbb{N}$, $k \geq 4$

③ Inductive step: $(k+1)$ $\quad 3^{(k+1)} > 2(k+1)^2+3(k+1)$

We want:

$3^{k+1} > 2(k+1)^2+3(k+1)$

$\iff 3^{k+1} > 2k^2+4k+2+3k+3$

$\iff 3^{k+1} > 2k^2+7k+5$

$3^{k+1} = 3^k 3 > 3(2k^2+3k) \iff$

$\iff 3^{k+1} > 6k^2+9k$

$\iff 6k^2+9k > 2k^2+7k+5$

$\iff 4k^2+2k-5 > 0$

$\iff 3^{k+1} > 6k^2+9k$

$\iff 3^{k+1} > 2k^2+7k+5$

$\iff 3^{k+1} > 2k^2+4k+2+3k+3$

$\iff 3^{k+1} > 2(k+1)^2+3(k+1)$

PoM Notes

1/4/1 $(A \times B) \times C \ni ((x,y),z)$

$A \times (B \times C) \ni (x,(y,z))$

$A \times B \times C \ni (x,y,z)$ <- triples

Defining as a set; $(x,y,z) = \{\{x\},\{x,y\},\{x,y,z\}\}$

$(x,x,x) = \{\{x\}\}$

1/4/7 S is a set w/ n elements then 2^n is the number of subsets

1) Induction

2) Power ~~sets~~

$\mathcal{P}(S)$ $\begin{cases} \emptyset \\ \{a\},\{b\},\{c\} \\ \{a,b\},\{a,c\},\{b,c\} \\ \{a,b,c\} \end{cases}$ $\left.\begin{matrix}\binom{3}{0} \\ \binom{3}{1} \\ \binom{3}{2} \\ \binom{3}{3}\end{matrix}\right\}$ $\begin{matrix} 1 \\ +3 \\ +3 \\ +1 \end{matrix}$

$2^0 = 1$

$2^1 = 1 \quad 1$

$2^2 = 1 \quad 2 \quad 1$

$2^3 = 1 \quad 3 \quad 3 \quad 1$

$2^4 = 1 \quad 4 \quad 6 \quad 4 \quad 1$

S let ~~LS~~ $\mathcal{P}(S \cup \{n\}) = \mathcal{P}(S) \cup$?

$\mathcal{P}(S)$ and ? has the same number of elements & none in common

Cantor's Theorem

There is no bijection between a set and its power set

CoM Notes

4.55 $\bigcup_{i\in I} A_i$ and $\bigcap_{i\in I} A_i$ $A_1=\{a,b,c,d\}$, $A_2=\{d,e,f\}$

$$\bigcup_{i\in I} A_i=\{a,b,c,d,e,f\}$$

$$\bigcap_{i\in I} A_i=\{d\}$$

$\bigcup_{i\in I} A_i=\{x \mid x\in A_i \text{ for some } i\in I\}$ Definition of families of sets

$\bigcap_{i\in I} A_i=\{x \mid x\in A_i \text{ for all } i\in I\}$

Definition of Power set

$$\boxed{X\in \mathcal{P}(A) \iff X\subseteq A}$$

4.56 Find $\bigcup_{i\in N} I_n$ and $\bigcap_{i\in N} I_n$ a) $I_n=[0,\frac{1}{n}]$
b) $I_n=(0,\frac{1}{n})$