10/13 Notes

4.48

$S = T$ iff $\mathcal{P}(S) = \mathcal{P}(T)$

$\Rightarrow$

let $\{x\}$ be a set s.t. $\{x\} \in \mathcal{P}(S)$

then $\{x\} \subseteq S$, By def of power set, $\mathcal{P}(T) = \{B \mid B \subseteq T\}$

and By Assumption $\{x\} \subseteq S \subseteq T$

$\Rightarrow \{x\} \subseteq T$

$\Leftrightarrow \{x\} \in \mathcal{P}(T)$, By def

$\Leftarrow$ It suffices to show $S \subseteq T \Leftrightarrow \mathcal{P}(S) \subseteq \mathcal{P}(T)$

Let $x \in S$, then $\{x\} \in \mathcal{P}(S)$

Since $\mathcal{P}(S) \subseteq \mathcal{P}(T)$

$\{x\} \in \mathcal{P}(T)$

$\Leftrightarrow \{x\} \subseteq T$,

$\Leftrightarrow x \in T$, By def of Powerset.

4.50 $\quad \mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$

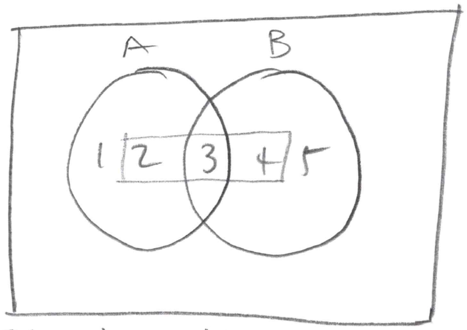
$\{2, 3, 4\} \in \mathcal{P}(A \cup B)$

But

$\{2, 3, 4\} \notin \mathcal{P}(A) \cup \mathcal{P}(B)$

this is disprove by counter example.

Visual



$\mathcal{P}(A \cup B)$ contains sets that are neither in $\mathcal{P}(A)$ or $\mathcal{P}(B)$

Salvaged!

$\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$

Let $\{x\} \in \mathcal{P}(A) \cup \mathcal{P}(B)$

$\iff \{x\} \in \mathcal{P}(A)$ or $\mathcal{P}(B)$

$\iff$ $X \subseteq A$ or $X \subseteq B$, By def of power set

$\Rightarrow$ $X \subseteq A \cup B$

$\iff \{x\} \in \mathcal{P}(A \cup B)$

4.51 $\mathcal{P}(A \setminus B) = \mathcal{P}(A) \setminus \mathcal{P}(B)$

$\Rightarrow$

Let $\{x\} \in \mathcal{P}(A \setminus B)$

$\Leftrightarrow \{x\} \subseteq A \setminus B$

$\Rightarrow \{x\} \subseteq A$ and $\{x\} \not\subseteq B$

$\Leftrightarrow \{x\} \in \mathcal{P}(A)$ and $\{x\} \notin \mathcal{P}(B)$, By def of power set

$\Leftrightarrow \{x\} \in \mathcal{P}(A) \setminus \mathcal{P}(B)$

$\Leftarrow$

The converse is False

$\mathcal{P}(A) \setminus \mathcal{P}(B) \subseteq \mathcal{P}(A \setminus B)$

therefore the proof is False

But can be salvaged!

$\boxed{\mathcal{P}(A \setminus B) \subseteq \mathcal{P}(A) \setminus \mathcal{P}(B)}$

Reasons:

Try counting elements "Zooming out"

if $B \subseteq A$

A has 5 elements

B has 2 elements

elements of $\mathcal{P}(A \setminus B) = 2^3 = 8$

elements of $\mathcal{P}(A) = 2^5 = 32$

elements of $\mathcal{P}(B) = 2^2 = 4$

elements of $\mathcal{P}(A) \setminus \mathcal{P}(B) = 28$

therefore

$\mathcal{P}(A \setminus B) \neq \mathcal{P}(A) \setminus \mathcal{P}(B)$

Try looking deeper "Zooming in"

If $\{x\} \subseteq B$ then $\{x\} \not\subseteq A \setminus B$

$\{x\} \subseteq A$ and $\{x\} \not\subseteq B$

$\Rightarrow \{x\} \subseteq A \setminus B$

$\forall x \in \{x\} : x \in A$, $\exists x \in \{x\} : x \notin B$

$\Rightarrow \forall x \in \{x\} : x \in A$ and $x \notin B$

This is False! Example, Problem 4.50 visual.

4.55

$\bigcup_{i\in I} A_i$ and $\bigcap_{i\in I} A_i$

$A_1 = \{a, b, c, d\}$, $A_2 = \{d, e, f\}$

$\bigcup_{i\in I} A_i = A_1 \cup A_2 = \{a, b, c, d, e, f\}$

$\bigcap_{i\in I} A_i = A_1 \cap A_2 = \{d\}$

def of: $\bigcup_{i\in I} A_i = \{x \mid x \in A_i, \exists i \in I\}$

$\bigcap_{i\in I} A_i = \{x \mid x \in A_i, \forall i \in I\}$

Index set , A_i , $i \in I$

$\mathcal{A} = \{A_i \mid i \in I\}$

Set of sets , "Family of sets"

Attempt problem 4.56

Find: $\bigcup_{i\in \alpha} I_m$

$\bigcap_{i\in \alpha} I_m$

(a) $I_m = [0, \frac{1}{m}]$

(b) $I_m = (0, \frac{1}{m})$