S 1.1 Show that $\sqrt{5}$ is not a rational number

Suppose $\sqrt{5}$ is a rational number

$\sqrt{5} = \frac{a}{b}$ → Assume that "a" is a multiple of 5

$5 = \frac{a^2}{b^2}$

$\boxed{a = 5n}$

$5b^2 = (5n)^2$

$5b^2 = 25n^2$

$b^2 = 5n^2$

$\boxed{b = 5n}$

Contradiction that it's a common factor of 5

$5b^2 = a^2$

prime numbers

$a = p_1 p_2 p_3 \cdots p_n$

$a^2 = p_1^2 p_2^2 p_3^2 \cdots p_n^2$

$5 | a^2 \Rightarrow 5 | a$

$5 | a^2 \Rightarrow \exists k \in \mathbb{N} : a^2 = 5k$

① Prime factorization

② Contrapositive

If $5 \nmid a \Rightarrow 5 \nmid a^2$

Case $a = 5k+2$

$a^2 = 25k^2 + 20k + 4 \Rightarrow 5 \nmid a^2$

$\Rightarrow (5 \nmid a \Rightarrow 5 \nmid a^2)$

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S 2.4 claim $\bigcup_{n \in \mathbb{N}} [0, \frac{1}{n}) = [0, 1)$ $\quad I_n = [0, \frac{1}{n})$

1) $[0,1) \subseteq \bigcup_{n \in \mathbb{N}} [0, \frac{1}{n})$

pf If $x \in [0,1) = I_1$, then there is a $n \in \mathbb{N}$ s.t. $x \in I_n$, thus $x \in \bigcup_{n \in \mathbb{N}} I_n = \bigcup_{n \in \mathbb{N}} [0, \frac{1}{n})$

2) $\bigcup_{n \in \mathbb{N}} [0, \frac{1}{n}) \subseteq [0,1)$

If $x \in \bigcup_{n \in \mathbb{N}} [0, \frac{1}{n})$ there is a $n \in \mathbb{N}$ s.t. $x \in [0, \frac{1}{n}) \subseteq [0,1)$

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4.49 $P(A \cap B) = P(A) \cap P(B)$

Let $X \in P(A \cap B)$

$\Leftrightarrow X \subseteq A \cap B$

$\Leftrightarrow X \subseteq A$ and $X \subseteq B$

$\Leftrightarrow X \in P(A)$ and $X \in P(B)$

$\Leftrightarrow X \in P(A) \cap P(B)$

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4.42 $M$ is a finite set $k \notin M$

① $P(M \cup \{k\}) = P(M) \cup \{B \mid B = A \cup \{k\} \wedge A \in P(M)\}$

② $\# P(M) = \# B$

③ $P(M) \cap B = \emptyset$

$M$ has $n$ elements

ind. If $\# P(M) = 2^n$

$\# (M \cup \{k\}) = n+1$

4.59 $\bigcup_{n\in\mathbb{N}} F_n = \mathbb{R}$

$\bigcap_{n\in\mathbb{N}} F_n = \emptyset$

$F_1 = (-1,0] \cup [0,1)$

$F_2 = (-2,-1] \cup [1,2)$

$F_3 = (-3,-2] \cup [2,3)$

$F_n = (-n, -(n-1)] \cup [n-1, n) \quad n \in \mathbb{N}$

$(-\infty, 0] \cup [0, \infty)$

$\bigcup_{n\in\mathbb{N}} F_n \subseteq \mathbb{R}$

"$\subseteq$" $x \in \bigcup_{n\in\mathbb{N}} F_n \Rightarrow x \in (-\infty, 0] \cup [0, \infty) \subseteq \mathbb{R}$

"$\supseteq$" $x \in \mathbb{R} \subseteq (-\infty, 0] \cup [0, \infty) \subseteq \bigcup_{n\in\mathbb{N}} F_n$

$\boxed{\begin{array}{l} x \in \mathbb{R} \text{ s.t. } x = k.d_1 d_2 d_3 \ldots \text{ decimal expansion} \\ x \in F_{k+1} \text{ if } k \geq 0 \\ x \in F_{-k+1} \text{ if } k < 0 \end{array}}$