Here is our system of four equations:

$$h_0^2 + h_1^2 + h_2^2 + h_3^2 = 1$$
 (1)

$$h_0 h_2 + h_1 h_3 = 0 \tag{2}$$

$$h_0 - h_1 + h_2 - h_3 = 0 \tag{3}$$

$$h_1 - 2h_2 + 3h_3 = 0 \tag{4}$$

We first solve (3) and (4) for  $h_2$  and  $h_3$ .

$$h_2 = -h_0 + h_1 + h_3 \tag{5}$$

Substituting (5) in the last equation yields

$$h_1 - 2(-h_0 + h_1 + h_3) + 3h_3 = 0$$
 (6)

and thus

$$h_3 = h_1 - 2h_0 \tag{7}$$

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Going back to (5) we obtain:

$$h_2 = 2h_1 - 3h_0 \tag{8}$$

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Next we substitute (7) and (8) in (2) and solve for  $h_1$ :

$$h_0(2h_1-3h_0)+h_1(h_1-2h_0)=0$$
 (9)

This yields

$$h_1^2 = 3h_0^2 \tag{10}$$

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Discarding one of the two solutions we obtain

$$h_1 = \sqrt{3}h_0 \tag{11}$$

Finally we substitute (7), (8) in (1):

$$h_0^2 + h_1^2 + (2h_1 - 3h_0)^2 + (h_1 - 2h_0)^2 = 1$$
 (12)

and obtain

$$14h_0^2 + 6h_1^2 - 16h_0h_1 = 1 \tag{13}$$

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Using (11) we get:

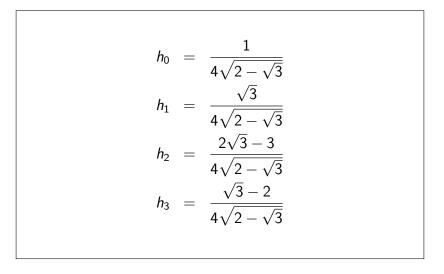
$$14h_0^2 + 18h_0^2 - 16\sqrt{3}h_0^2 = 1 \tag{14}$$

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Solving for  $h_0$  (again discarding one of the two solutions) we obtain:

$$h_0 = \frac{1}{4\sqrt{2-\sqrt{3}}}$$
 (15)

Backsolving we can compute all 4 coefficients:



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## Finally note that

$$\begin{array}{rcl} h_0 + h_1 + h_2 + h_3 \\ = & \displaystyle \frac{1}{4\sqrt{2 - \sqrt{3}}} (1 + \sqrt{3} + 2\sqrt{3} - 3 + \sqrt{3} - 2) \\ = & \displaystyle \frac{1}{4\sqrt{2 - \sqrt{3}}} (-4 + 4\sqrt{3}) = \displaystyle \frac{\sqrt{3} - 1}{\sqrt{2 - \sqrt{3}}} \\ = & \displaystyle \frac{(\sqrt{3} - 1)(\sqrt{2 + \sqrt{3}})}{(\sqrt{2 - \sqrt{3}})(\sqrt{2 + \sqrt{3}})} \\ = & \displaystyle \sqrt{4 - 2\sqrt{3}}\sqrt{2 + \sqrt{3}} \\ = & \displaystyle \sqrt{2}\sqrt{(2 - \sqrt{3})(2 + \sqrt{3})} = \sqrt{2} \end{array}$$

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