More on Accumulation Points

Problem 1
Every infinite bounded set of real numbers has at least one accumulation point.

Problem 2
Characterize all infinite sets that have no accumulation points.[1]

The next tasks in this section explore the relationship between the limit of a converging sequence and accumulation points of its range. Recall that the range of a sequence (a_n) is the set $\{a_n \mid n \in \mathbb{N}\}$.

Problem 3

1. Find a converging sequence whose range has an accumulation point.
2. Find a converging sequence whose range has no accumulation points.
3. Show that the range of a converging sequence has at most one accumulation point.

Problem 4
Suppose the sequence (a_n) is bounded and satisfies the condition that $a_m \neq a_n$ for all $m \neq n \in \mathbb{N}$. Show: If its range $\{a_n \mid n \in \mathbb{N}\}$ has exactly one accumulation point a, then (a_n) converges to a.

[1] This means to conjecture a theorem of the form: An infinite set has no accumulation points if and only if ...

Problem 5

Find a sequence (a_n) for which the set of accumulation points of its range is the set of real numbers.