5.1.#11

The system has 6 equilibrium points:

In[9]:=
```
Solve[{x (-x - y + 40) == 0, y (-x^2 - y^2 + 2500) == 0}]
```

Out[9]= $\{\{x \to 0, y \to -50\}, \{x \to 0, y \to 50\}, \{x \to 40, y \to 0\},$
$\{x \to 5(4 - \sqrt{34}), y \to 5(4 + \sqrt{34})\}, \{x \to 5(4 + \sqrt{34}), y \to 5(4 - \sqrt{34})\}, \{x \to 0, y \to 0\}\}$

In[10]:=
```
N[%]
```

Out[10]=

$\{\{x \to 0., y \to -50.\}, \{x \to 0., y \to 50.\}, \{x \to 40., y \to 0.\},$
$\{x \to -9.15476, y \to 49.1548\}, \{x \to 49.1548, y \to -9.15476\}, \{x \to 0., y \to 0.\}\}$

In[20]:=
```
Sqrt[850]
```

Out[20]=

$5\sqrt{34}$

Only 3 equilibrium points are in the first quadrant: {0,50},{40,0},{0,0}:

In[11]:=
```
Linearization[x (-x - y + 40), y (-x^2 - y^2 + 2500) , x, y, {0, 0}]
```

The point {0, 0} is an equilibrium point.

The linearized system at {0, 0} has the form Y'(t)= $\begin{pmatrix} 40 & 0 \\ 0 & 2500 \end{pmatrix} \cdot$ Y(t).

The eigenvalues are $\lambda_1 = 2500.$ and $\lambda_2 = 40.$.

In[12]:=
```
Linearization[x (-x - y + 40), y (-x^2 - y^2 + 2500) , x, y, {0, 50}]
```

The point {0, 50} is an equilibrium point.

The linearized system at {0, 50} has the form Y'(t)= $\begin{pmatrix} -10 & 0 \\ 0 & -5000 \end{pmatrix} \cdot$ Y(t).

The eigenvalues are $\lambda_1 = -5000.$ and $\lambda_2 = -10.$.

In[13]:=
```
Linearization[x (-x - y + 40), y (-x^2 - y^2 + 2500) , x, y, {40, 0}]
```

The point {40, 0} is an equilibrium point.

The linearized system at {40, 0} has the form Y'(t)= $\begin{pmatrix} -40 & -40 \\ 0 & 900 \end{pmatrix} \cdot$ Y(t).

The eigenvalues are $\lambda_1 = 900.$ and $\lambda_2 = -40.$.

Here is the linearization for the equilibrium points not in the first quadrant:

In[15]:=
```
Linearization[x (-x - y + 40), y (-x^2 - y^2 + 2500) , x, y, {0, -50}]
```

The point {0, −50} is an equilibrium point.

The linearized system at {0, −50} has the form Y'(t)= $\begin{pmatrix} 90 & 0 \\ 0 & -5000 \end{pmatrix} \cdot$ Y(t).

The eigenvalues are $\lambda_1 = -5000.$ and $\lambda_2 = 90.$.

In[16]:=
```
Linearization[x (-x - y + 40), y (-x^2 - y^2 + 2500) , x, y, {5 (4 - √34), 5 (4 + √34)}]
```

The point $\{5\,(4-\sqrt{34}),\ 5\,(4+\sqrt{34})\}$ is an equilibrium point.

The linearized system at $\{5\,(4-\sqrt{34}),\ 5\,(4+\sqrt{34})\}$ has the form Y'(t)= $\begin{pmatrix} 5\,(-4+\sqrt{34}) & 5\,(-4+\sqrt{34}) \\ 900 & -100\,(25+4\sqrt{34}) \end{pmatrix}\cdot$ Y(t).

The eigenvalues are $\lambda_1 = -4834.08$ and $\lambda_2 = 10.856$.

In[17]:=
```
Linearization[x (-x - y + 40), y (-x^2 - y^2 + 2500) , x, y, {5 (4 + √34), 5 (4 - √34)}]
```

The point $\{5\,(4+\sqrt{34}),\ 5\,(4-\sqrt{34})\}$ is an equilibrium point.

The linearized system at $\{5\,(4+\sqrt{34}),\ 5\,(4-\sqrt{34})\}$ has the form Y'(t)= $\begin{pmatrix} -5\,(4+\sqrt{34}) & -5\,(4+\sqrt{34}) \\ 900 & 100\,(-25+4\sqrt{34}) \end{pmatrix}\cdot$ Y(t).

The eigenvalues are $\lambda_1 = -108.387 + 201.819\,i$ and $\lambda_2 = -108.387 - 201.819\,i$.