Last Time:

Cooling pizza

oven is 500°. Take pizza out of the oven and it starts cooling

How long do we have to wait until the pizza can be eaten?

Physics: Newton's Law of Cooling

the rate of cooling is proportional to the difference b/w the object's temperature and the ambient temperature

$C(t)$ temp of pizza at time t ($^\circ F$)
t minutes
C^* ambient temperature $= 70^\circ$

$$\frac{dC(t)}{dt} \sim C^* - C(t)$$

$$\Leftrightarrow \boxed{\frac{dC(t)}{dt} = k\left(C^* - C(t)\right)}$$

Newton's Law of Cooling

$C(0) = 500$
need one more piece of information
$C(1) = 480$

edible temperature: $\bar{C} = 110^\circ$

$C(1) = 480$

edible temperature; $\bar{C} = 110^\circ$

$$\begin{cases} C' = k(70 - C) \\ C(0) = 500 \\ C(1) = 480 \end{cases}$$

this is a linear DEQ!

$$c' + kC = 70k \qquad \int \cdot e^{kt}$$

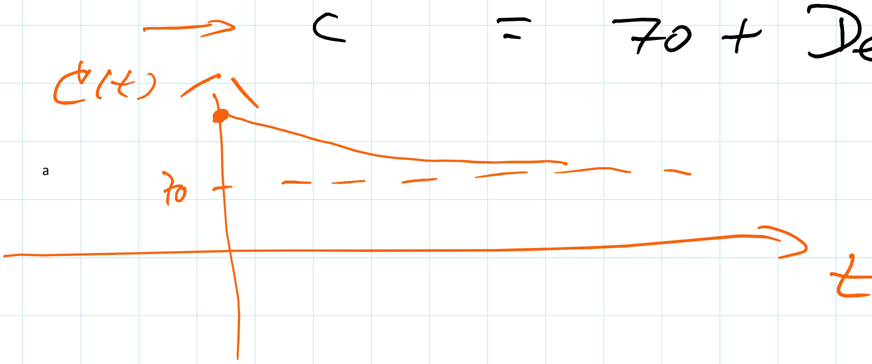
$$c'e^{kt} + ke^{kt}C = 70ke^{kt}$$

$$(c \cdot e^{kt})' = 70ke^{kt}$$

$$Ce^{kt} = \int 70ke^{kt}\,dt$$

$$Ce^{kt} = 70e^{kt} + D$$

$$C = 70 + De^{-kt}$$



a

Continuing:

$$C(0) = 500 = 70 + De^{-k\cdot 0} = 70 + D$$

$$\Rightarrow D = 430$$

Lazy Man's Rule

$$C(t) = 70 + 430e^{-kt}$$

We also know: $C(1) = 480$

$$480 = C(1) = 70 + 430e^{-k\cdot 1}$$

$$410 = 430e^{-k}$$

$$\frac{410}{430} = e^{-k}$$

$$\ln\frac{41}{43} = -k$$

$$\Rightarrow k = -\ln\frac{41}{43} = \ln\frac{43}{41} \quad (>0)$$

$$C(t) = 70 + 430e^{-\ln\frac{43}{41}\cdot t}$$

Let t_0 be such that $C(t_0) = \bar{C} = 110$

$$110 = C(t_0) = 70 + 430e^{-\ln\frac{43}{41}\cdot t_0}$$

$$40 = 430e^{-\ln\frac{43}{41}\cdot t_0}$$

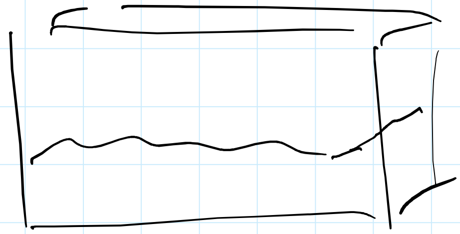
$$\frac{40}{430} = e^{-\ln\frac{43}{41}\cdot t_0}$$

$$\ln\frac{4}{43} = -\ln\frac{43}{41}\cdot t_0$$

$$t_0 = -\frac{\ln\frac{4}{43}}{\ln\frac{43}{41}} = \frac{\ln\frac{43}{4}}{\ln\frac{43}{41}} \approx 50 \text{ minutes}$$

2nd application:

Water tank problems



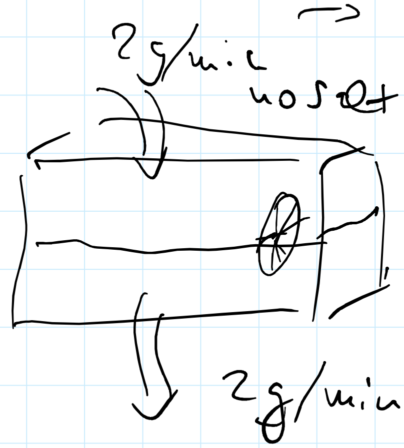
$W(t)$ amount of ~~water~~ solution in tank (in g) at time t

$\rightarrow W(0) = 20$

$S(t)$ will be the amount of salt in tank at time t (in lbs)

t time (in minutes)

$\rightarrow S(0) = 30$



$S(0) = 30$

rate of change of salt in the tank (in $\frac{lbs}{min}$) $\rightarrow$

$$S'(t) = -\text{ flowrate} \times \text{concentration}$$

$$= -2\,\frac{g}{min} \times \frac{S(t)\text{ lbs}}{20g} \quad \left(\frac{lbs}{min}\right)$$

$$= -2 \cdot \frac{1}{20} S(t)$$

initial value problem
$$\left[\begin{array}{l} S(0) = 30 \\ S' = -\frac{1}{10} S(t) \end{array}\right.$$

sep ✓

lin ✓

$$S' + \frac{1}{10} S(t) = 0 \quad \Big|\, e^{\frac{t}{10}}$$

$$S' e^{\frac{t}{10}} + \frac{1}{10} e^{\frac{t}{10}} S(t) = 0$$

$$s'e^{\frac{t}{10}} + \frac{1}{10}e^{\frac{t}{10}}S(t) = 0$$

$$\left(S\cdot e^{\frac{t}{10}}\right)' = 0$$

$$S e^{\frac{t}{10}} = C$$

$$S(t) = Ce^{-\frac{t}{10}}$$

Since $S(0)=30$, $C=30$

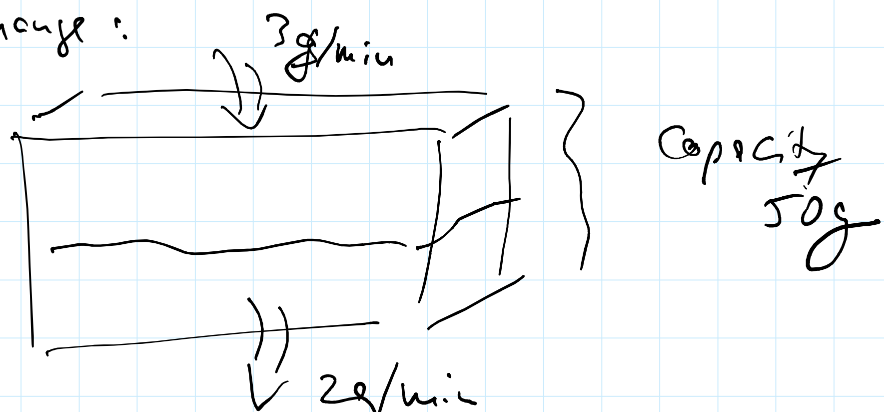
so

$$\boxed{S(t) = 30e^{-\frac{t}{10}}}$$

Let's change:

$W(0)=20$

$S(0)=30$



$\underline{\underline{Q}}$ How much salt is in the tank when the tank is full?

$$\left[\begin{array}{l} W(0)=20 \\ \frac{dW}{dt} = 1 = 3-2 \end{array}\right. \qquad W(t) = 20+t$$

$$\frac{dS}{dt} = -\text{ flow rate} \times \text{concentration}$$

$$= -\ 2\,\frac{g}{min}\cdot\frac{S(t)}{W(t)}\ \frac{lbs}{g}$$

$$= -\ 2\cdot\frac{S(t)}{20+t}$$

IVP

$$S(0) = 30$$

$$S' = -2\frac{S}{20+t} = -\frac{2}{20+t}\cdot S$$

lin.

$$S' + \frac{2}{20+t}S = 0$$

find an integrating factor:

$$p(t) = \frac{2}{20+t}$$

(no |·| necessary since $t>0$)

$$\int p(t)\,dt = 2\ln(20+t)$$

$$I(t) = e^{\int p(t)\,dt} = e^{2\ln(20+t)} = e^{\ln((20+t)^2)}$$

$$= (20+t)^2$$

$$(20+t)^2 S'(t) + 2(20+t)\cdot S = 0$$

check ✓

$$\left(S(t)\cdot(20+t)^2\right)' = 0$$

$$S(t)\cdot(20+t)^2 = C$$

$$\boxed{S(t) = \frac{C}{(20+t)^2}}$$

$$30 = S(0) = \frac{C}{20^2} = \frac{C}{400}$$

$$\Rightarrow C = 30\cdot 400 = 12{,}000$$

$$\boxed{S(t) = \frac{12{,}000}{(20+t)^2}}$$

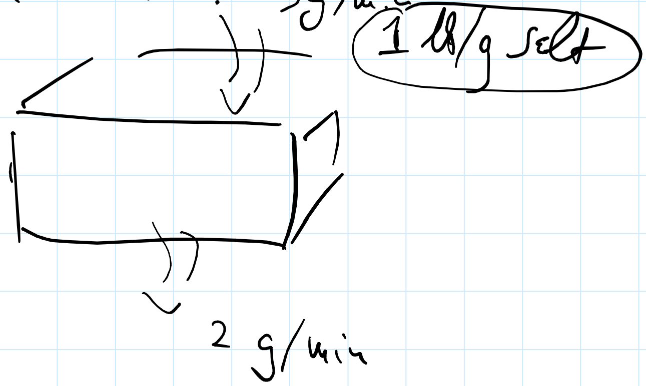
Tank will be full when $t = 30$

$$S(t) = \frac{12{,}000}{50^2} = \frac{12{,}000}{2{,}500}$$

Change every min : 3 g/min

$V(0) = 20$

$S(0) = 30$



$V(t) = 20 + t$

$$S'(t) = + \text{flow in} \times \text{conc} - \text{flowout} \times \text{conc.}$$

$$= 3\frac{\cancel{g}}{\min} \cdot 1 \frac{\text{lbs}}{\cancel{g}} - 2\frac{\cancel{g}}{\min} \frac{S(t)}{20+t} \frac{\text{lbs}}{\cancel{g}}$$

rate of change of salt $\left(\frac{\text{lbs}}{\min}\right)$

$\frac{\text{lbs}}{\min}$ $\frac{\text{lbs}}{\min}$

$$\begin{cases} S'(t) = 3 - \dfrac{2}{20+t} S \\ S(0) = 30 \end{cases}$$ IVP

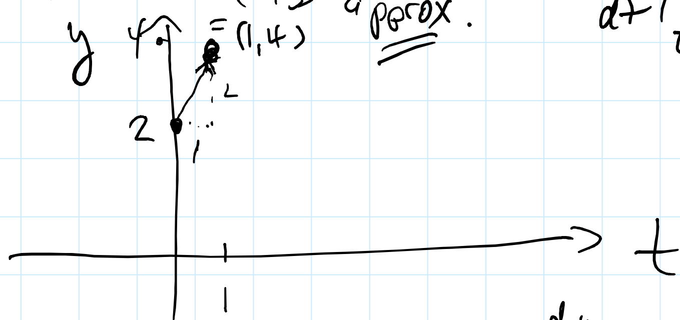
lin. DEQ

Solve! $S'(t) + \frac{2}{20+t} S = 3$

What can we do .eqs ?

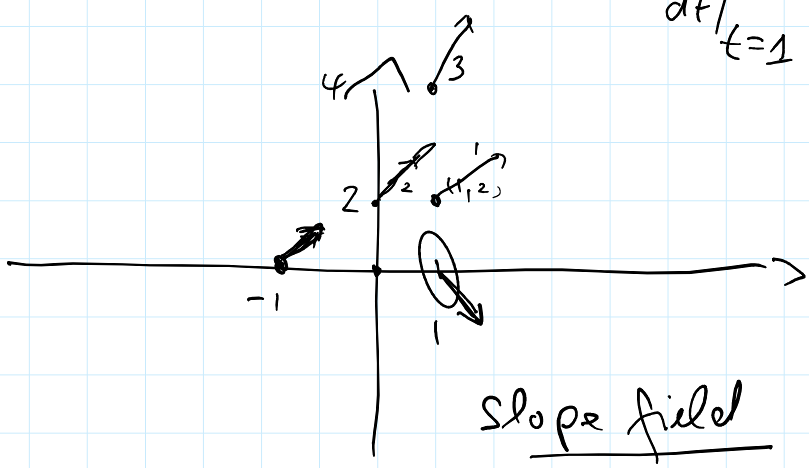
IVP $\begin{cases} y' = y - t \\ y(0) = 2 \end{cases}$

$(t,y) = (1,4)$ approx.



$$\left.\frac{dy}{dt}\right|_{t=0} = y - t = 2 - 0 = 2$$

$$\left.\frac{dy}{dt}\right|_{t=1} = y - t = 4 - 1 = 3$$



$$\left.\frac{dy}{dt}\right|_{t=-1} = 0 - (-1) = 1$$

Slope field

(direction field = normalised slope field vectors)

↪ sketch solutions without "calculus"