Last time: Toricelli's Law

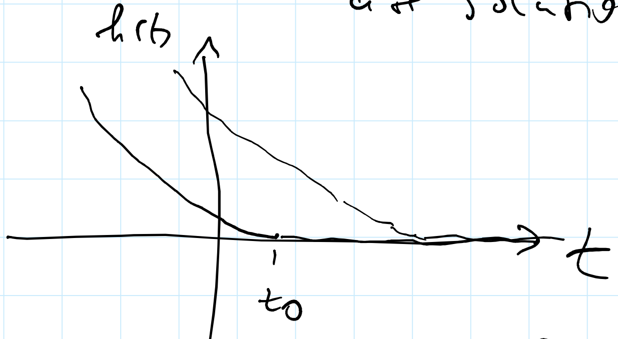
$$\frac{dh}{dt} = -2h^{1/2}$$

$h(t)$ height of water in the bucket at time t

We want to check:

$$h(t) = \begin{cases} (t-t_0)^2 & t \le t_0 \\ 0 & t > t_0 \end{cases}$$

are solutions to DEQ for all t_0



$$\frac{dh}{dt} = \begin{cases} 2(t-t_0) & \text{if } t < t_0 \\ 0 & t > t_0 \end{cases}$$

Both expressions are $= 0$.

$\frac{dh}{dt} = -2h^{1/2}$?

$$\underline{R.S.} = -2\left((t-t_0)^2\right)^{1/2} \quad (t \le t_0)$$

$$= -2(t-t_0)$$

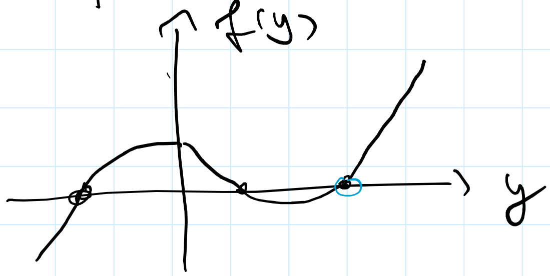
$$RS = 0 \quad (t > t_0)$$

Autonomous DEQ's

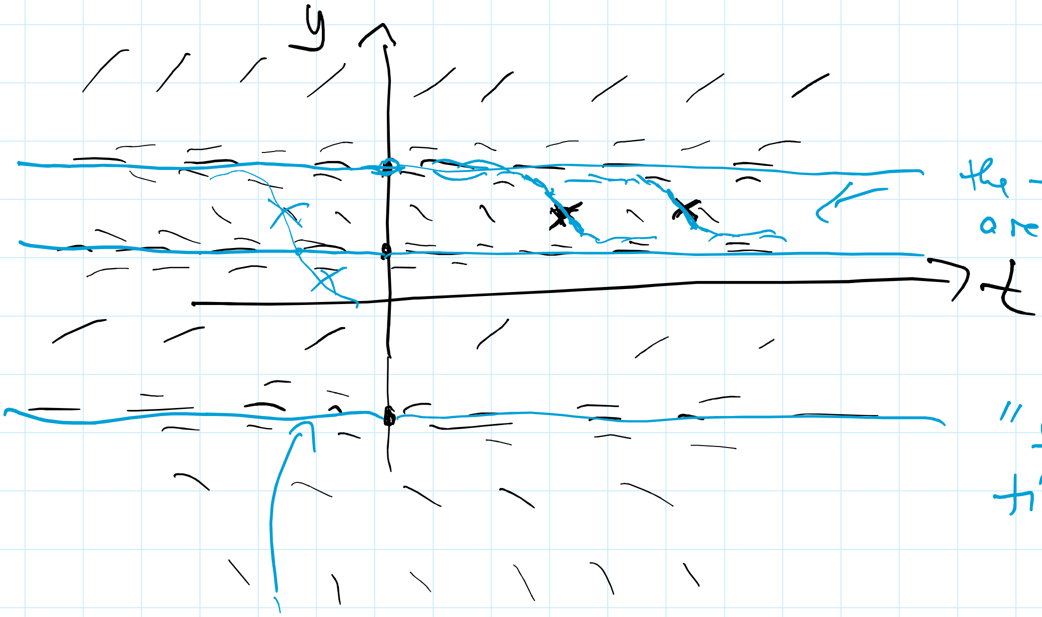
R.S. of DEQ only depends on y (not on t)

$$y' = f(y)$$

Ex. from Q2



slope field:

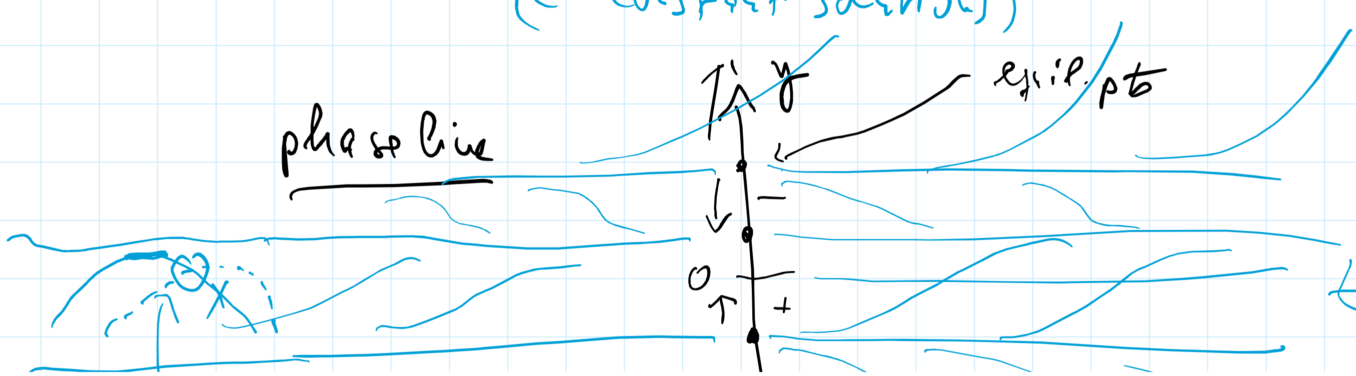


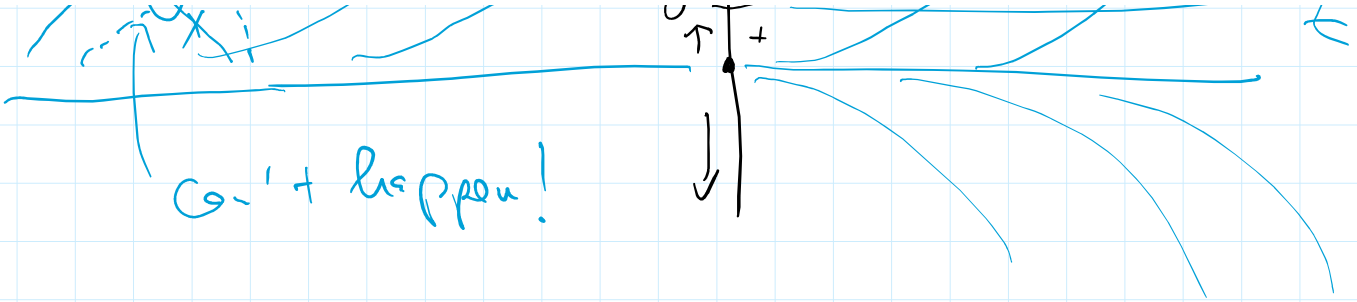
the two solutions are horizontal shifts of each other

"solutions are time-shift invariant"

"equilibrium solutions"
(= constant solutions)

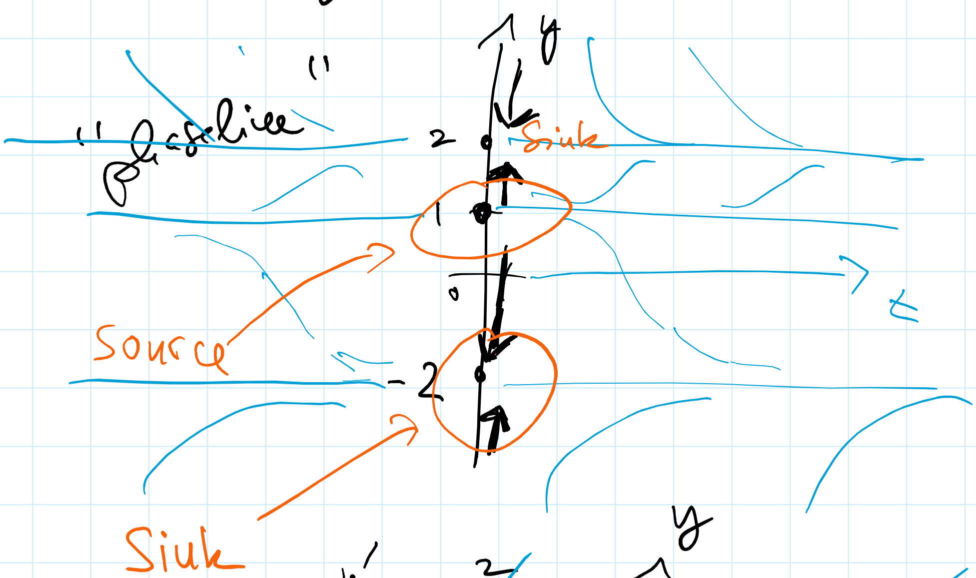
phase line





More examples:

$$y' = (1-y)(y^2-4)$$

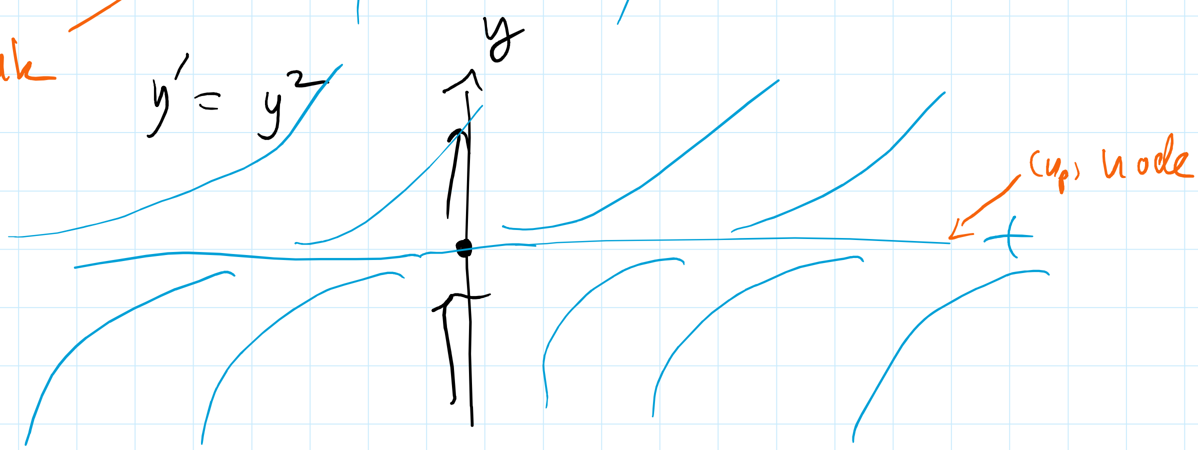


If $y < 2$, $(1-y) > 0$
$(y^2-4) > 0$

$-2 < y < 1$ $(1-y) > 0$
$(y^2-4) < 0$

$1 < y < 2$ $(1-y) < 0$
$(y^2-4) < 0$

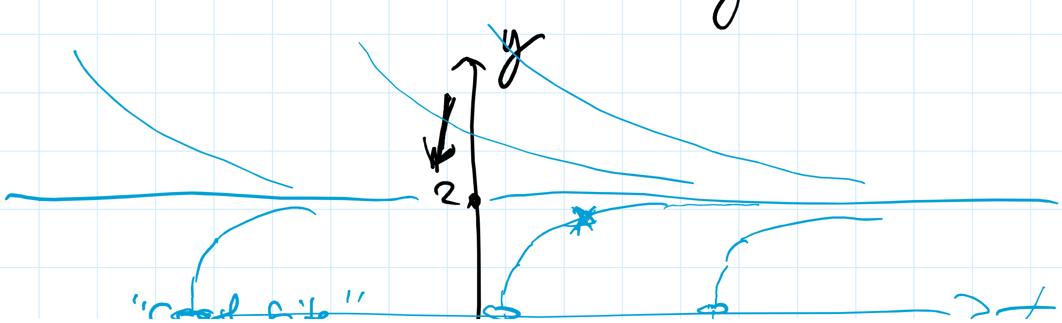
$$y' = y^2$$

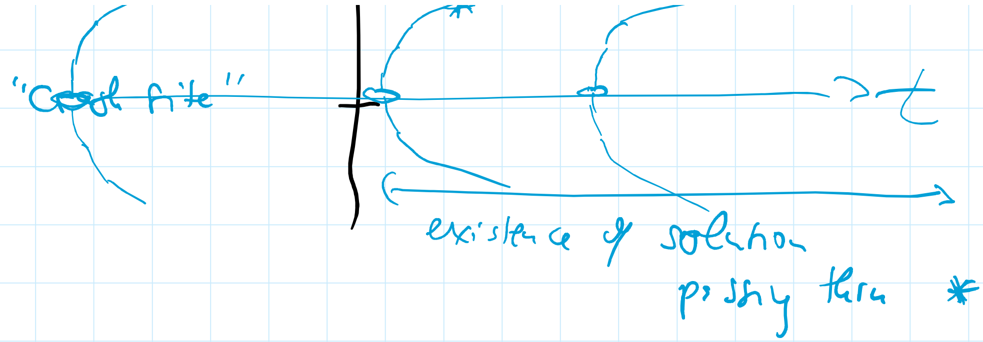


Example

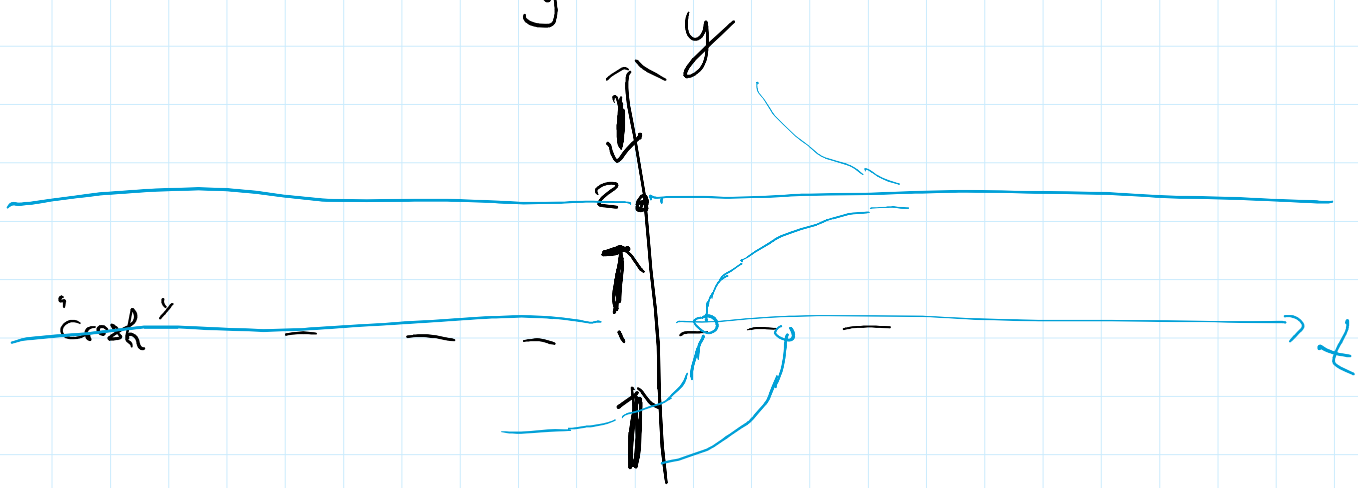
$$y' = \frac{2-y}{y}$$

autonomous DEQ





$$y' = \frac{2-y}{y^2}$$



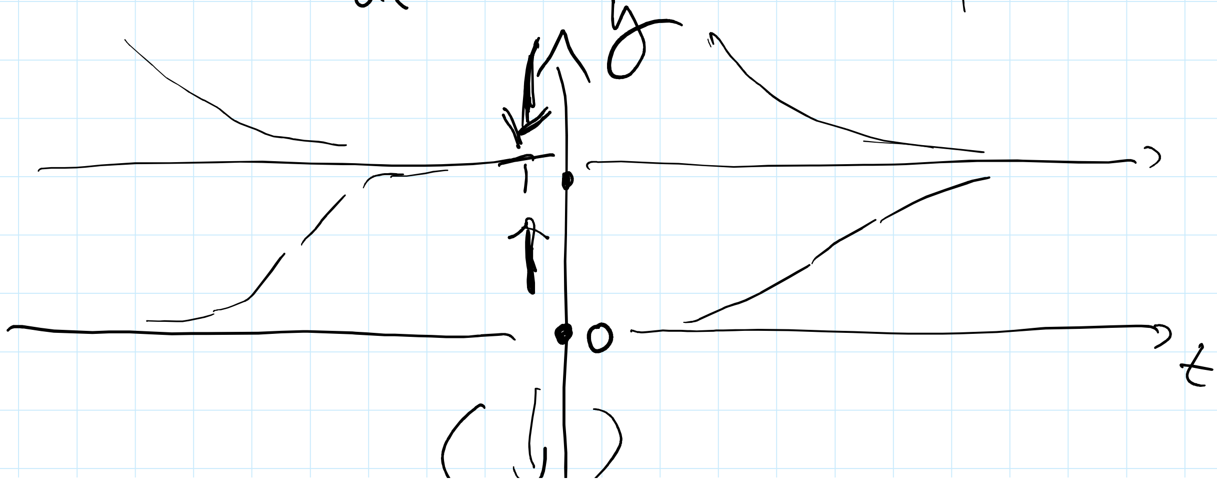
Logistic Equation

growth with a population ceiling

$P(t)$ population at time t

$$\frac{dP(t)}{dt} = k \cdot P(t)\left(1 - \frac{P(t)}{T}\right)$$

T population ceiling



(J1)