Monday, February 9, 2026 14:51

Test 1 next Wednesday
→ will cover Chapter 1
→ Preparation: read the book, Do the homework problems
→ for Monday: prepare questions!

Autonomous DEQ $y' = f(y)$ (no dependance on t)

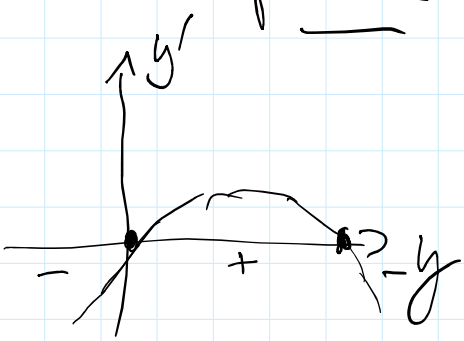
Logistic growth

$$y' = ky\left(1 - \frac{y}{N}\right)$$

$y' = ky - \frac{k}{N}y^2$

k growth factor
N population ceiling

phaseline



equilibrium = pts where $y' = 0$

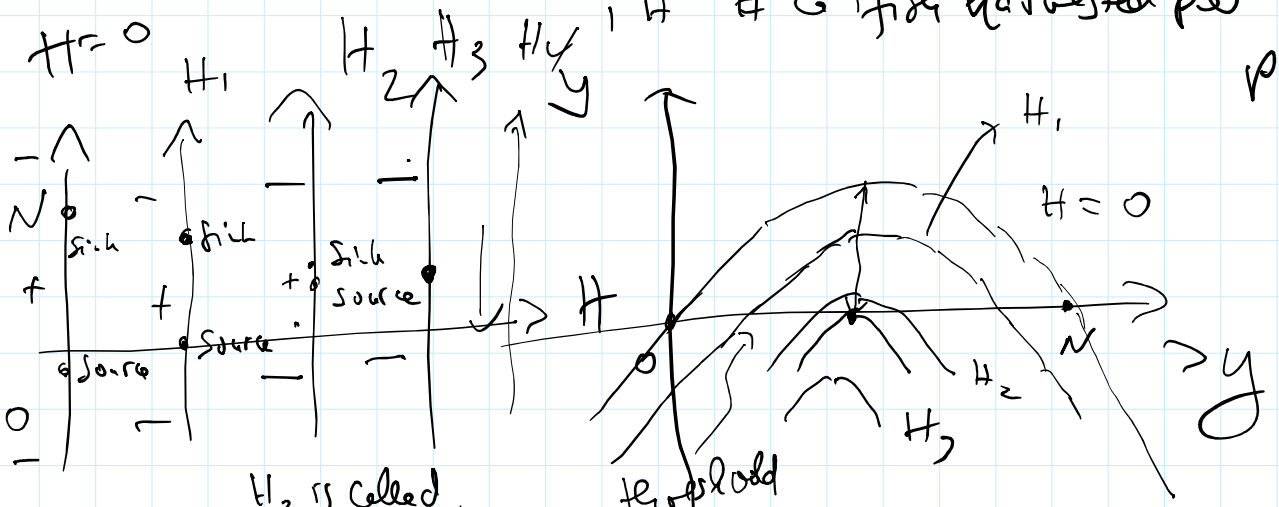
logistic growth with harvesting

$$y' = ky\left(1 - \frac{y}{N}\right) - H$$

H # of fish harvested per time -

N

H = # of fish harvested per time-period



H_3 is called the critical harvesting rate

threshold

2 equil pts

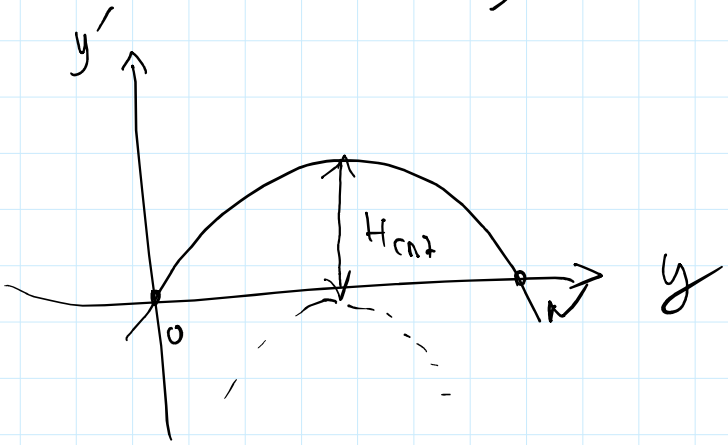
sink source

1 equil. pt

at the critical harvesting rate H_3, a bifurcation occurs

A bifurcation value is a parameter value where the number and/or character of the equilibrium points changes

→ what is the critical harvesting rate?



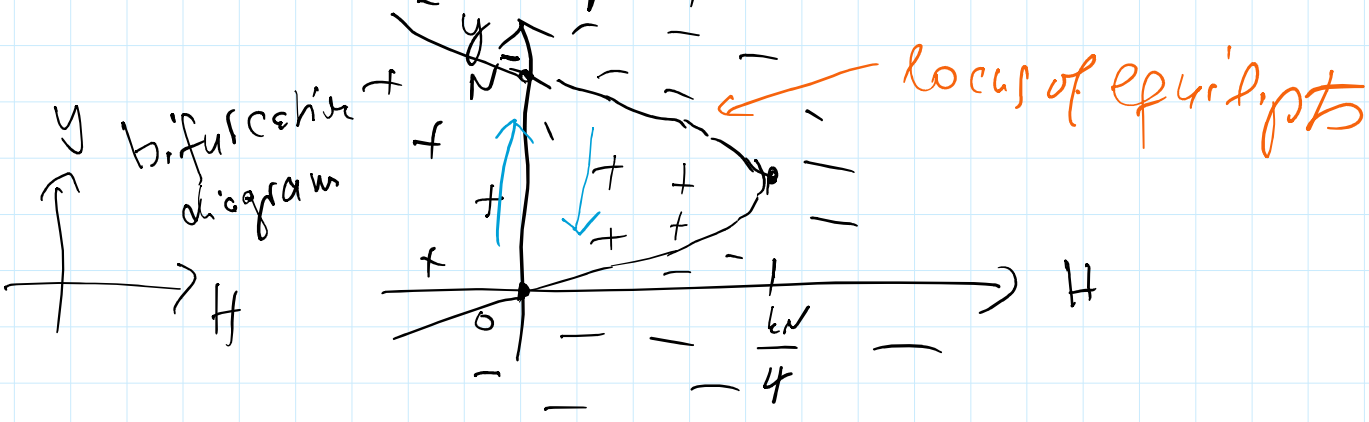
$$y'\Big|_{y=\frac{N}{2}} = y'\left(\frac{N}{2}\right) = k\frac{N}{2}\left(1-\frac{\frac{N}{2}}{N}\right)$$

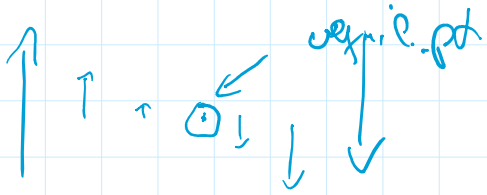
$$= \frac{kN}{2}\,\frac{1}{2} = \frac{kN}{4}$$

$\frac{kN}{2} \cdot \frac{1}{2} = \left(\frac{kN}{4}\right)$

software: Stella

→ Locus of Equilibrium Points





in our case:

$$0 = ky\left(1 - \frac{y}{N}\right) - H$$

$$\Leftrightarrow H = ky - \frac{k}{N}y^2$$

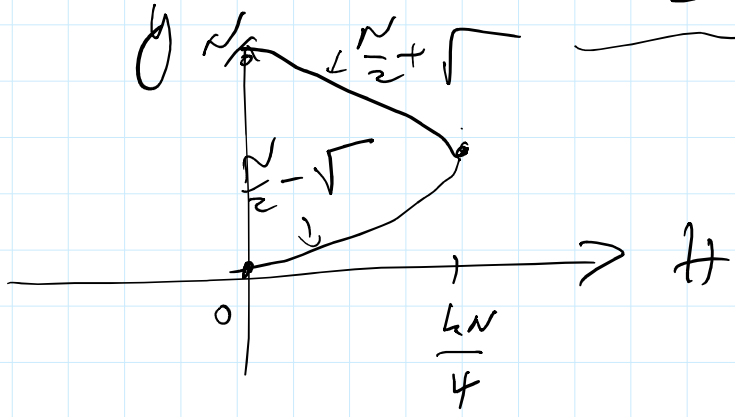
Solve for y:

$$\frac{k}{N}y^2 - ky + H = 0 \quad | \cdot \frac{N}{k}$$

quadratic equation in y

$$y^2 - Ny + \frac{NH}{k} = 0$$

$$y = \frac{N}{2} \pm \sqrt{\frac{N^2}{4} - \frac{NH}{k}}$$

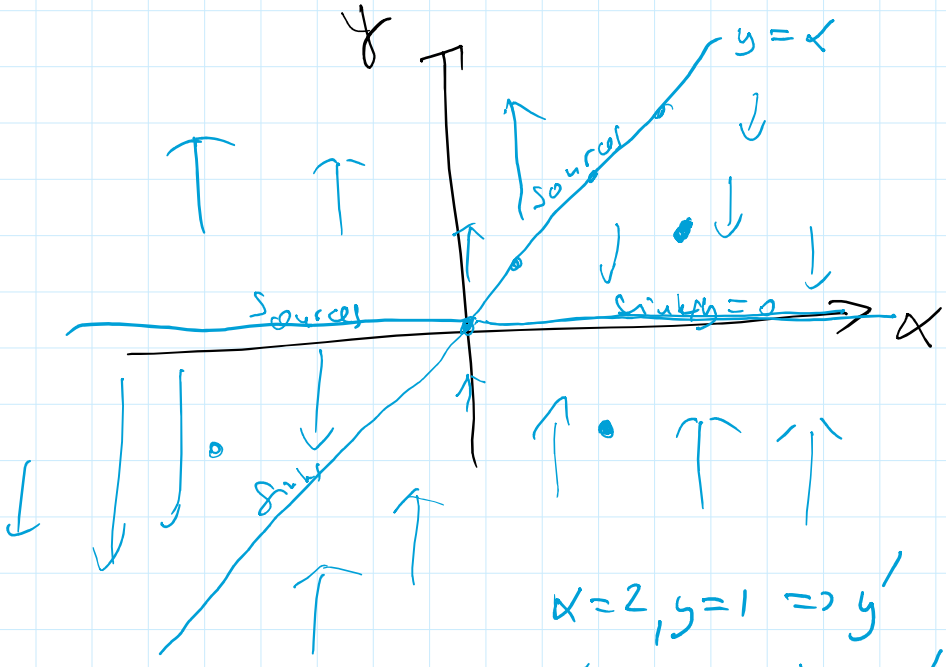


$$\frac{N^2}{4} - \frac{NH}{k} = 0$$

$$\Leftrightarrow H = \frac{kN}{4}$$

Example 2

$$y' = y(y - \alpha)$$ α parameter



Locus of equil. pts

$$0 = y(y - \alpha)$$

$$\Leftrightarrow y = 0 \text{ or } y = \alpha$$

$\alpha = 2, y = 1 \Rightarrow y' < 0$

$\alpha = 1, y = -1 \Rightarrow y' > 0$

$\alpha = 1, y = -1 \Rightarrow y' > 0$

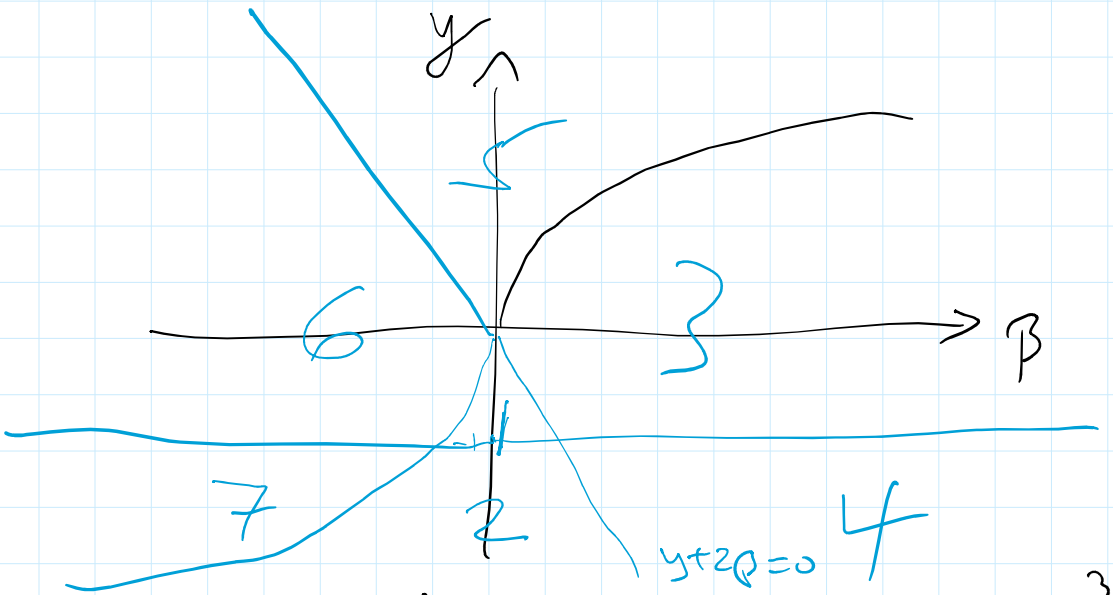
$\alpha = -2, y = -1 < 0$

at $\alpha = 0$ bifurcation occurs

↳ 1 equil pt : up - node

Example

$$y' = (y+2\beta)(y^3-\beta)(y+1)$$



loc. of equil pts: $(y+2\beta)(y^3-\beta)(y+1) = 0$

$\Leftrightarrow y+2\beta = 0$ or $y^3 = \beta$

or $y+1 = 0$

$\Leftrightarrow y = -2\beta$ or $y = \sqrt[3]{\beta}$ or $y = -1$

Systems of Differential Equations

2 dependent variables (x, y)

2 dependent variables (x, y)

1 independent variable (t)

Predator – Prey – Model

foxes $F(t)$ (predator)

rabbits $R(t)$ (prey)

only rabbits: $R'(t) = 2R\left(1 - \frac{R}{2}\right)$ log. growth model

only foxes (no rabbits) $F'(t) = -F$

how to model encounters b/w foxes and rabbits

$enc(F, R)$

$enc(R, F) = 0$ if $R = 0$ or $F = 0$

$\frac{\partial enc}{\partial R}(R, F) > 0$

$\frac{\partial enc}{\partial F}(R, F) > 0$

easiest function: $enc(R, F) = R \cdot F$

$$R' = 2R\left(1 - \frac{R}{2}\right) - 1.2RF$$

$$F' = -F + 0.9RF$$

autonomous

equilibrium point?

autonomous DEQ's

equilibrium pt!

$\Leftrightarrow R' = 0$ and $F' = 0$