2/16

Wednesday, February 11, 2026 15:01

#16 $\quad \dfrac{dy}{dt} = \dfrac{t}{y-2} \qquad y(-1) = 0$

$(y-2)\,dy = t\,dt$ sep. DEQ.

$\dfrac{y^2}{2} - 2y = \dfrac{t^2}{2} + C$ done if we were to solve in implicit form

to compute C: $\dfrac{0^2}{2} - 0 = \dfrac{(-1)^2}{2} + C$

$\Rightarrow C = -\dfrac{1}{2}$

Sol.: $\dfrac{y^2}{2} - 2y = \dfrac{t^2}{2} - \dfrac{1}{2}$

if we seek a solution in explicit form:

$\dfrac{1}{2}y^2 - 2y - \left(\dfrac{t^2}{2} - c\right) = 0$

(quadratic equation)

$y^2 - 4y - (t^2 - D) = 0$

$y = 2 \cancel{\pm} \sqrt{4 - (t^2 - D)}$

$y(-1) = 0 \Rightarrow y < 2$

$y = 2 - \sqrt{4 - (t^2 - D)}$

$0 = 2 - \sqrt{4 - ((-1)^2 - D)}$

$0 = 2 - \sqrt{3 + D}$

$2 = \sqrt{3 + D} \Rightarrow D = 1$

solution: $y = 2 - \sqrt{4 - (t^2 - 1)}$

$= 2 - \sqrt{5 - t^2}$

solution is valid for

$-\sqrt{5} < t < \sqrt{5}$

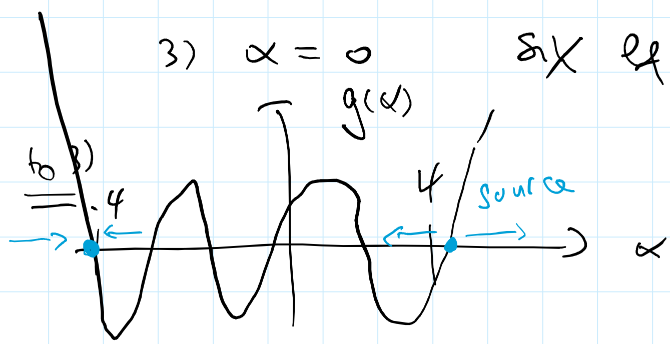
1.7 #16

$$\frac{dy}{dt} = g(y) + \alpha$$

1) $\alpha \le -4$ one sink no other equil.

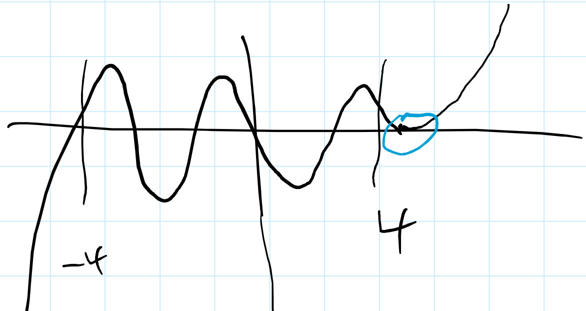
2) $\alpha \ge 4$ —— " ——

3) $\alpha = 0$ six equilibria

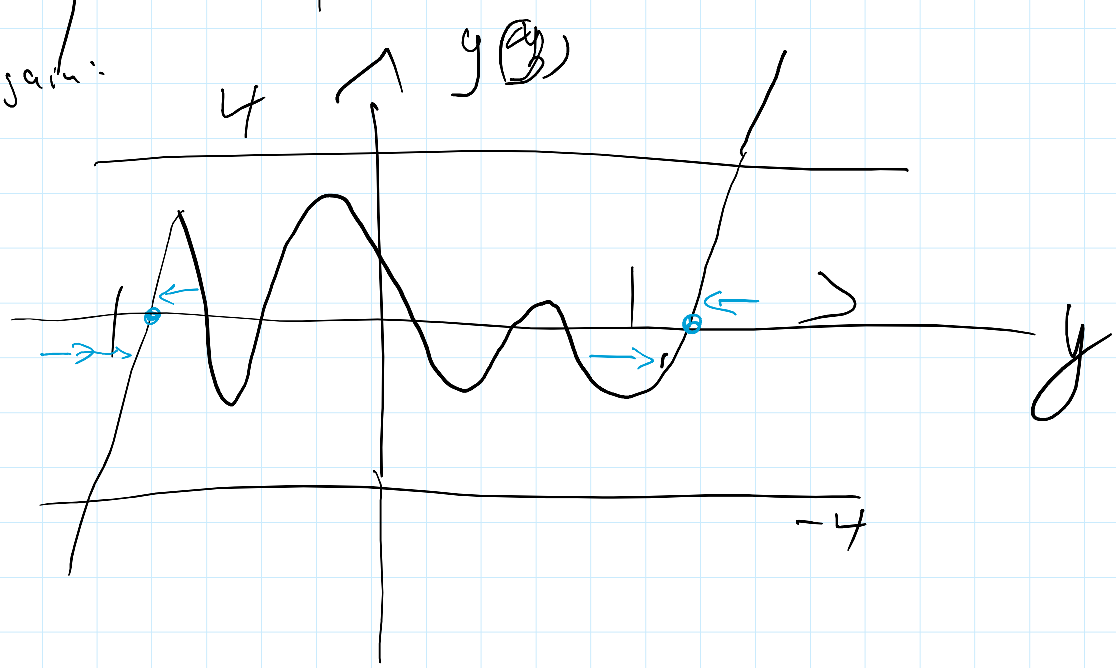


4 possibilities

or



again:



$g(y) + \alpha$

1.9 #19

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For what values of α is it possible to find explicit solutions to

$$\frac{dy}{dt} = \alpha t y + 4e^{-t^2} \ ?$$

$$y' - \alpha t y = 4e^{-t^2}$$

$$p(t) = -\alpha t$$

$$I(t) = e^{\int p(t)dt} = e^{-\int \alpha t\,dt} = e^{-\alpha \frac{t^2}{2}} = e^{-\frac{\alpha}{2}t^2}$$

$$\underbrace{y' e^{-\frac{\alpha}{2}t^2} - \alpha t y e^{-\frac{\alpha}{2}t^2}} = 4e^{-(1+\frac{\alpha}{2})t^2}$$

$$\left(y e^{-\frac{\alpha}{2}t^2}\right)' = 4e^{-(1+\frac{\alpha}{2})t^2}$$

$$y e^{-\frac{\alpha}{2}t^2} = 4\int e^{-(1+\frac{\alpha}{2})t^2}dt$$

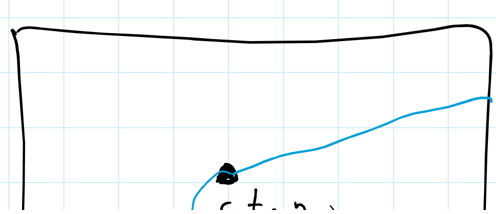
can find integral if $1+\frac{\alpha}{2} = 0$ and that's the only possibility

Ex & Uniqu

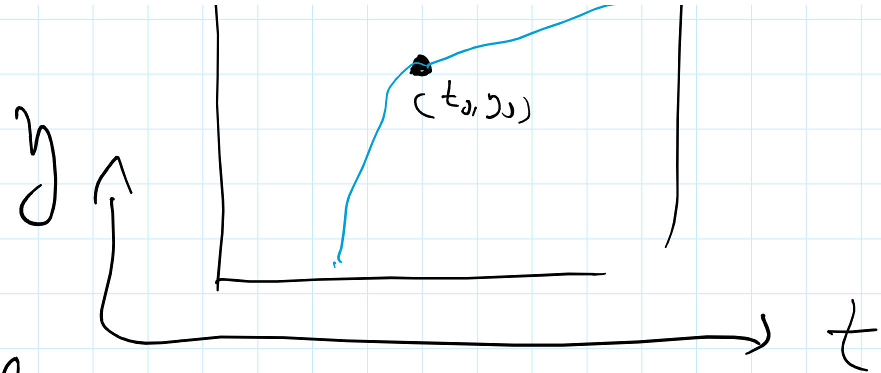
$$\frac{dy}{dt} = f(t,y)$$

If f, and $\frac{\partial f}{\partial y}$ are

R



$\frac{\partial f}{\partial y}$ are continuous on R



containing (t_0, y_0)

then the initial value problem

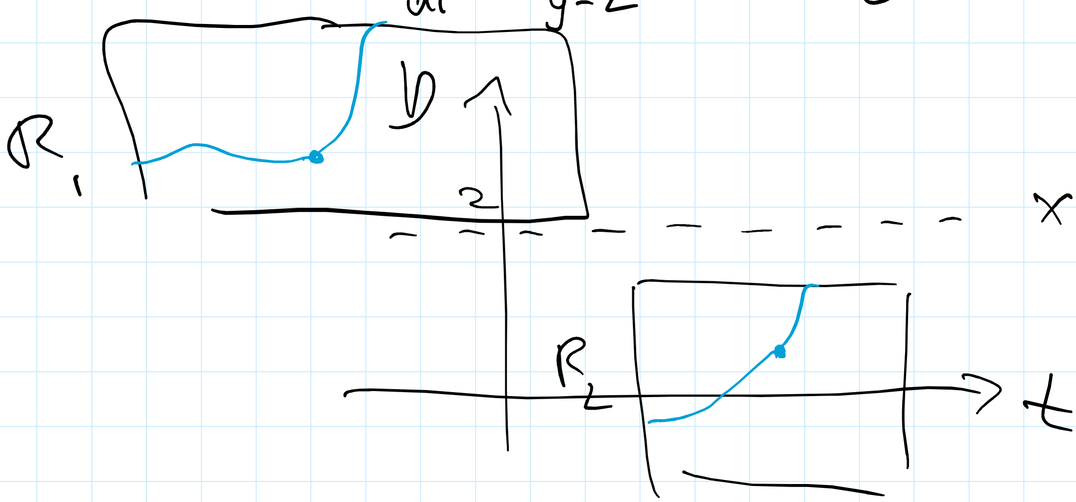
$$\frac{dy}{dt} = f(t,y) \qquad y(t_0) = y_0$$

$$\boxed{\frac{dy}{dt} = f(t,y)}$$

will have a unique solution as long as it stays in R

Ex

$$\frac{dy}{dt} = \frac{t}{y-2} = f(t,y)$$



f not cont. when $y=2$

If f is "nice" everywhere

then every solution builds a fence for all the other solutions

1.5

$$\frac{dy}{dt} = f(y) \qquad f \text{ is "nice"}$$

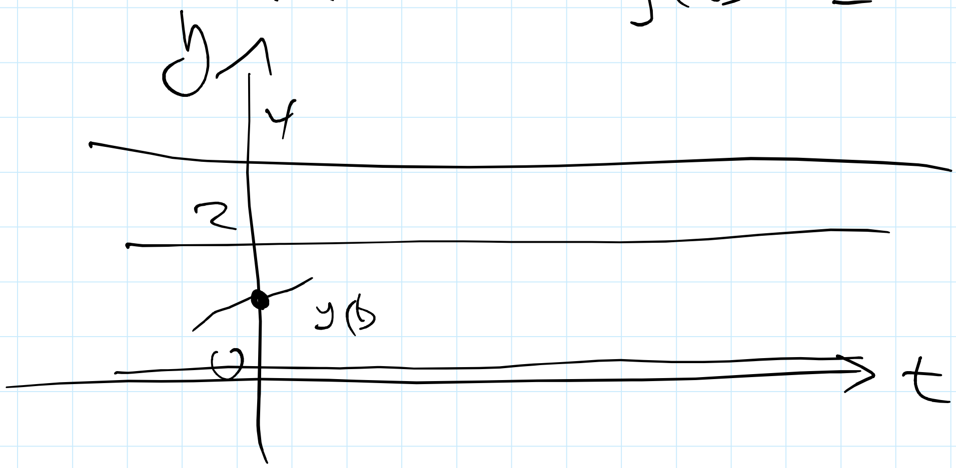
$$\frac{dy}{dt} = f(y) \qquad f \text{ is } \ldots$$

$y_1(t) = 4$ is a solution

$y_2(t) = 2$ — " —

$y_3(t) = 0$ — " —

init. condition: $y(0) = 1$



answer: $0 < y(t) < 2$ for all t

1.5. #11

$$\frac{dy}{dt} = \frac{y}{t^2}$$

$$f(t,y) = \frac{y}{t^2}$$

$$\underbrace{\frac{\partial f}{\partial y}(t,y) = \frac{1}{t^2}}_{\text{not continuous when } t=0}$$

rest is similar to the bucket problem.

Problem

$$\frac{dy}{dt} = 2\sqrt{|y|}$$

a) $y = 0$ is a solution

a) $y = 0$ is a solution

b) Find all solutions

c) Ex & Uniqueness Thm.?

(similar to bucket problem)

a) $y = 0$ ✓ L.S. $\frac{dy}{dt} = 0$

R.S. $= 2\sqrt{|y|} = 2\sqrt{0} = 0$ ✓

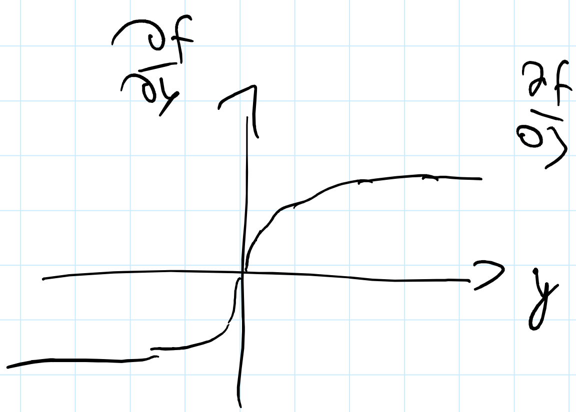
b) $$\frac{dy}{dt} = 2\sqrt{|y|}$$

$$\frac{dy}{|y|^{\frac{1}{2}}} = 2\,dt \qquad \underline{\text{Sep. DeQ}}$$

$f(y) = 2\sqrt{|y|}$ continuous for all y

$$\frac{\partial f}{\partial y} = \begin{cases} y^{-\frac{1}{2}} & \text{if } y > 0 \\ -(-y)^{-\frac{1}{2}} & \text{if } y < 0 \end{cases} \qquad f(y) = \begin{cases} 2\sqrt{y} & \text{if } y \geq 0 \\ 2\sqrt{-y} & \text{if } y < 0 \end{cases}$$

$$f(y) = \begin{matrix} 2y^{\frac{1}{2}} & (y \geq 0) \\ 2(-y)^{\frac{1}{2}} & (y < 0) \end{matrix}$$



$\frac{\partial f}{\partial y}$ not defined when $y = 0$

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$\frac{dy}{dt} = f(y)$ $\quad f$ cont. everywhere

f diff. — " —

$\frac{dy}{dt} = f(y)$ $\quad$ f cont. everywhere

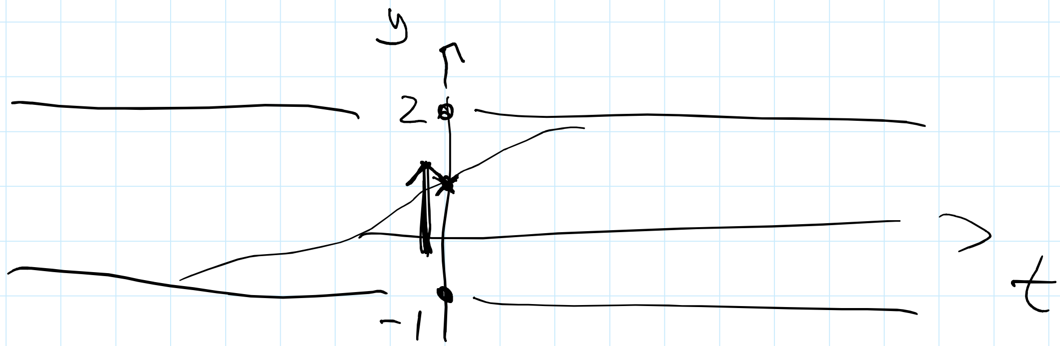
f diff. —||—

$f(-1) = f(2) = 0$

What if $y(t)$ with $y(0) = 1$

$f(-1) = 0 \Rightarrow y = -1$ is a const. solution

$f(2) = 0 \Rightarrow y = 2$ is a const. solution



$-1 < y(t) < 2$ for all t

b) $f(y) > 0$ for $-1 < y < 2$

Can additionally say:

$y(t)$ increasing for all t

$$\lim_{t \to \infty} y(t) = 2$$

$$\lim_{t \to -\infty} y(t) = -1$$

Problem

$$\frac{dy}{dt} = y^2 + 3y + \alpha$$

Draw bifurcation diagram!

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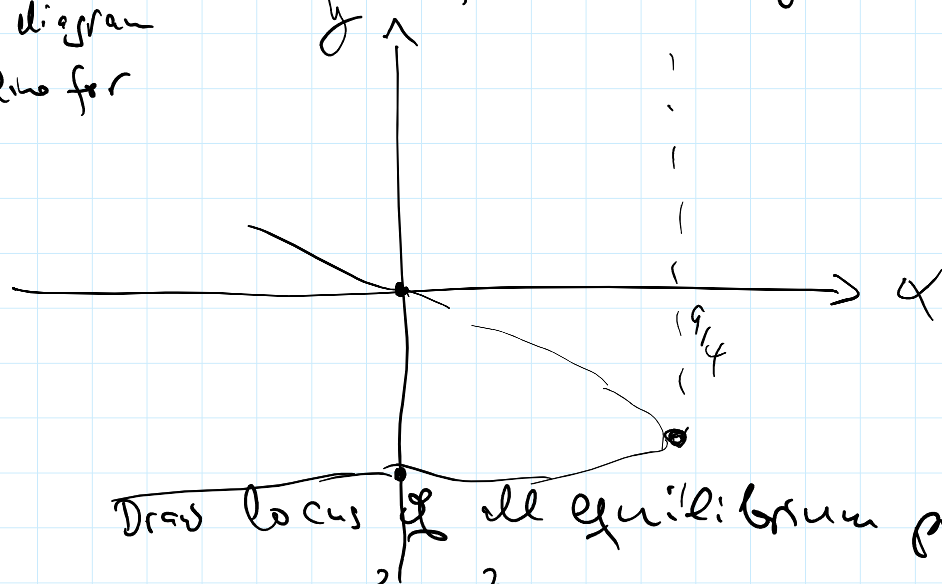
bifurcation diagram = a phaseline for every α



Draw locus of all equilibrium points!

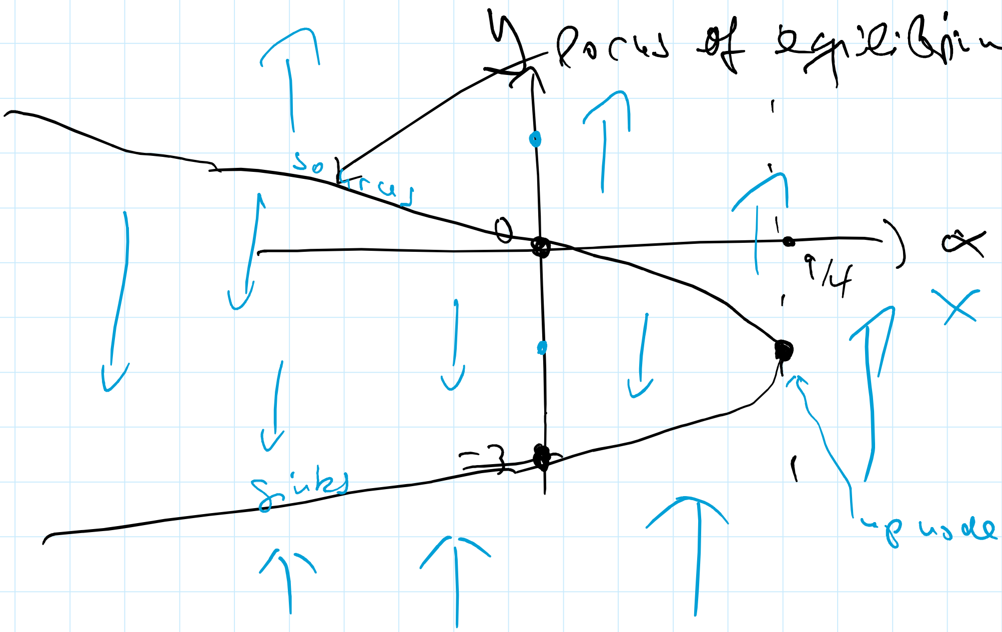
$$y^2 + 3y + \alpha = 0$$

$$\alpha = -y^2 - 3y$$

quadratic equation in y:

$$y = -\frac{3}{2} \pm \sqrt{\frac{9}{4} - \alpha}$$

for $\alpha = \frac{9}{4}$: $y = -\frac{3}{2}$



$\alpha = 0, y = 2$
$y' > 0$

$\alpha = 0 \; y = -2$
$y' < 0$

only bifurcation value:

our bifurcation value:

when $\alpha = \frac{9}{4}$

$\alpha < \frac{9}{4}$	to the left:	$\frac{\text{source}}{\text{sink}}$	2 eqn. pts
	at $\frac{9}{4}$:	up node	1. eqpn
	$\alpha > \frac{9}{4}$	0	0

Office Hours

M 3-4

T 1-3 &

$4^{30} - 5^{30}$