Test next W.
covers up to 5.1
(included)

Separatrices:

$$x' = -x$$
$$y' = -4x^3 + y$$

Linearize the system

only equil. pt is $(0,0)$

$$\mathrm{Jac} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ -12x^2 & 1 \end{pmatrix} \text{ at } (0,0)$$
$$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

eigenvalues are 1 and -1

$\hookrightarrow$ saddle

info carries over to the non-lin. system ✓

The non-linear is partially decoupled

$$x' = -x$$
$$\Rightarrow x = Ae^{-t}$$

$$y' = -4A^3e^{-3t} + y$$
$$\Leftrightarrow y' - y = -4A^3e^{-3t} \quad | \; e^{-t}$$
$$(ye^{-t})' = -4A^3e^{-4t}$$

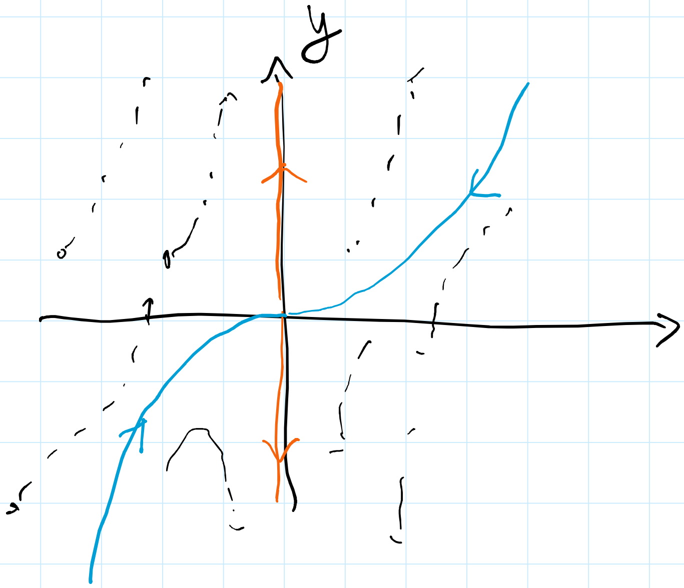
$$(ye^{-t})' = -4A^3e^{-4t}$$

$$ye^{-t} = A^3e^{-4t} + B$$

$$y = A^3e^{-3t} + Be^{t}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} Ae^{-t} \\ A^3e^{-3t} \end{pmatrix} + \begin{pmatrix} 0 \\ Be^{t} \end{pmatrix}$$

Set $A = 0$ $\qquad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ B \end{pmatrix} e^{t}$



Set $B = 0$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} Ae^{-t} \\ A^3e^{-3t} \end{pmatrix} = \begin{pmatrix} Ae^{-t} \\ (Ae^{-t})^3 \end{pmatrix}$$

$$\Leftrightarrow y = x^3$$

⟸ stable separatrix
can determine bases of attraction

Theorem (Poincare-Bendixson)

In $\mathbb{R}^2$ $\lim_{\substack{t\to\infty \\ (t\to-\infty)}} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$

is either ① an equilibrium point

② a limit cycle

③ or tends to infinity

3 or tends to infinity