5/6

Wednesday, May 6, 2026 13:48

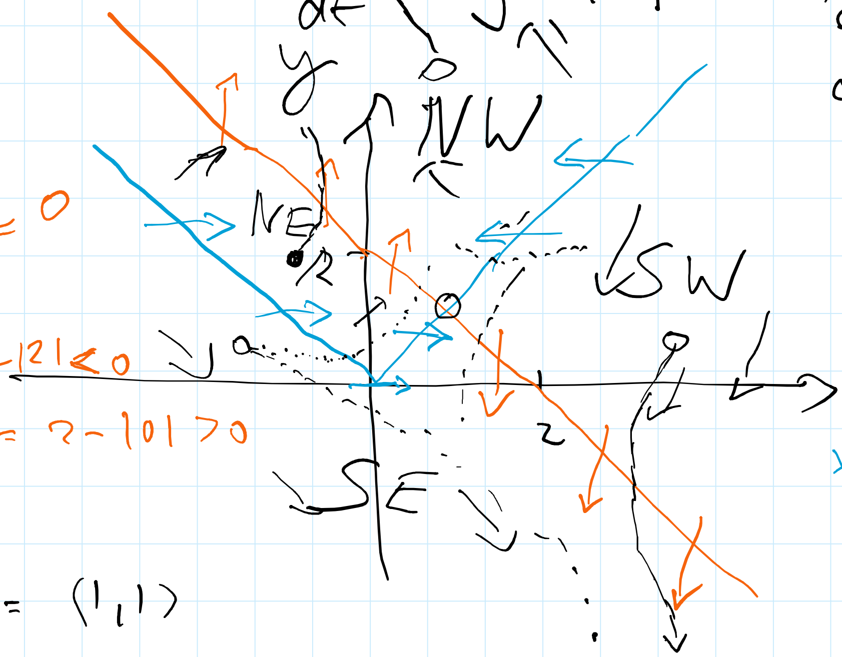
Test is comprehensive!

5.2 #2

$$\frac{dx}{dt} = 2 - x - y$$

$$\frac{dy}{dt} = y - |x|$$

Nullclines; find equil. pt, classify them



x-nullcline $2 - x - y = 0$

at $(2,0)$: $y' = 0 - |2| < 0$

at $(0,2)$: $y' = 2 - |0| > 0$

y-nullcline

$(2,2)$: $x' = 2 - x - y = 2 - 2 - 2 < 0$

$(0,0)$: $x' = 2 - 0 - 0 > 0$

equi. pts = $(1,1)$

equil. pt. analysis

$$Jac(1,1) = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$(\lambda+1)(\lambda-1) - 1 = 0$$

$$\lambda^2 - 1 - 1 = 0$$

$$\lambda^2 = 2$$

$$\lambda = \pm\sqrt{2}$$

saddle! ✓

Inca – Diff. Equations

Juca – Diff. Equations

~ 1930s

Complex eigenvalues Problem

$$x'' + 2x' + 8x = 0$$

write as a system

$$x' = v$$

$v = x'$

$$x'' = v' = -2v - 8x$$

$$\begin{pmatrix} x' \\ v' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -8 & -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ v \end{pmatrix}$$

!

eigenvalues:

$$\lambda(\lambda+2) + 8 = 0$$

$$\lambda^2 + 2\lambda + 8 = 0$$

$$(\lambda+1)^2 + 7 = 0$$

$$(\lambda+1)^2 = -7$$

$$\lambda + 1 = \pm\sqrt{7}\, i$$

$$\lambda = -1 \pm \sqrt{7} \cdot i$$

spiral sink

1 eigenvector for $\lambda = -1 + \sqrt{7}\, i$

$$A - \lambda \mathrm{Id} = \begin{pmatrix} 0 & 1 \\ -8 & -2 \end{pmatrix} - \begin{pmatrix} 1+\sqrt{7}i & 0 \\ 0 & 1+\sqrt{7}i \end{pmatrix}$$

$$= \begin{pmatrix} -1-\sqrt{7}i & 1 \\ -8 & -1-\sqrt{7}i \end{pmatrix}$$

eigenvector will satisfy:

$$-8x - (1+\sqrt{7}i)\,y = 0$$

$$(x \quad (1+\sqrt{7}i \;)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1+\sqrt{7}i \\ -8 \end{pmatrix}$$

one solution:

$$Y(t) = \begin{pmatrix} 1+\sqrt{7}i \\ -8 \end{pmatrix} e^{-t} (\cos\sqrt{7}t + i \sin\sqrt{7}t)$$

$$Y(t) = e^{-t}\left[\begin{pmatrix} \cos\sqrt{7}t - \sqrt{7}\sin\sqrt{7}t \\ -8\cos\sqrt{7}t \end{pmatrix} + i \begin{pmatrix} \sqrt{7}\cos\sqrt{7}t + \sin\sqrt{7}t \\ -8\sin\sqrt{7}t \end{pmatrix}\right]$$

general solution

$$Y(t) = Ae^{-t}\begin{pmatrix} \cos\sqrt{7}t - \sqrt{7}\sin\sqrt{7}t \\ -8\cos\sqrt{7}t \end{pmatrix} + Be^{-t}\begin{pmatrix} \sqrt{7}\cos\sqrt{7}t + \sin\sqrt{7}t \\ -8\sin\sqrt{7}t \end{pmatrix}$$

A, B constants

! no i in sight

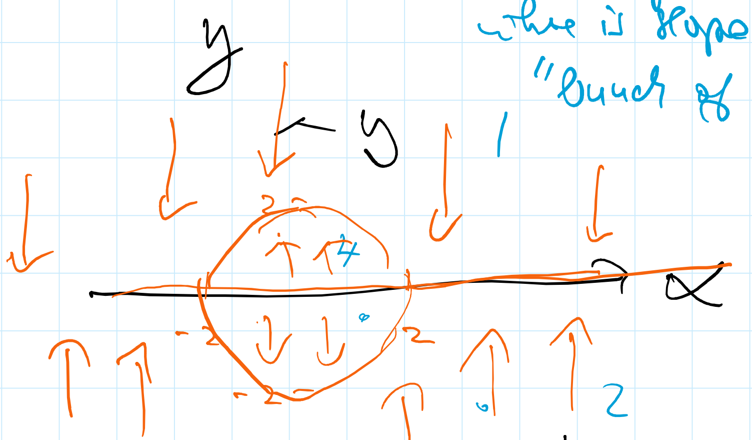
P5 Test 2

$$\frac{dy}{dt} = y(4 - \alpha^2 - y^2)$$

bifurcation diagram

where is slope pos & neg.

"bunch of phaselines"



where are the equil. pts.?

$\left(\frac{dy}{dt} = 0 \;?\right)$

$y = 0$ or $\alpha^2 + y^2 = 4$

$y = 0$ or $\alpha^2 + y^2 = 4$

in region 1 $(2,2)$ $\frac{dy}{dt} = 2(4 - 2^2 - 2^2) < 0$

region 2 $(-2, 2)$ $\frac{dy}{dt} = -2(4 - 2^2 - 2^2) > 0$

region 3 $(1, -1)$ $\frac{dy}{dt} = -1(4 - 1^2 - 1^2) < 0$

1 equil. pt		3 equil. pts		1 equil. pt
sink	-2	sink source sink	2	sink

bifurcations occur at $\alpha = -2$ and $\alpha = 2$