Homework 1 **Introduction to Analysis** **Fall 2000**

The problems are due on September 25.

Problem 1. Prove the following: If (x_n) is a sequence so that $\{x_n \mid n \in \mathbb{N}\}$ has **at least two** accumulation points, then (x_n) diverges.

Problem 2. A sequence (a_n) is called *proper*, if $a_n \neq a_m$ for all $n \neq m$. Show that a proper bounded sequence converges, if $\{a_n \mid n \in \mathbb{N}\}$ has **exactly one** accumulation point.

Problem 3. Let $\epsilon > 0$, and let (x_n) be a convergent sequence of real numbers. Show that there is a **Cauchy** sequence (r_n) of rational numbers satisfying $|x_n - r_n| \leq \epsilon$ for all $n \in \mathbb{N}$.