Homework 2 **Introduction to Analysis** **Fall 2000**

The problems are due on November 8.

Problem 1. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two continuous functions. Define

$$h(x) = \max\{f(x), g(x)\} \text{ for all } x \in \mathbb{R}.$$

Show that h is continuous on $\mathbb{R}$.

Problem 2. Show that every open set on the real line is the countable union of disjoint open intervals.

Problem 3. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function on $\mathbb{R}$ so that both $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$. Show that f is uniformly continuous on $\mathbb{R}$.

(We say that $\lim_{x\to\infty} f(x) = L$ if for all $\epsilon > 0$ there is an $x_0 \in \mathbb{R}$ such that for all $x \in \mathbb{R}$:

$$(x > x_0 \Rightarrow |f(x) - L| < \epsilon)\,.$$

The limit at $-\infty$ is defined analogously.)