No books, notes etc. are allowed.

Problem 1. Let (X, d) be a metric space.

1. Give the definition of "open set" in (X, d).
2. Define: x is an accumulation point of the set A in X.
3. Recall that a set is closed if its complement is open. Show that a set A is closed in X if and only if A contains all of its accumulation points in X.

Problem 2. Let $\mathcal{C}$ denote the continuous functions on the interval $[0, 1]$, equipped with the norm $\|f\| = \sup\limits_{0 \le x \le 1} |f(x)|$. Is the set

$$A = \{f \in \mathcal{C} \mid f(0) = f(1)\}$$

a closed set in $\mathcal{C}$? Is it open in $\mathcal{C}$? Explain carefully!

Problem 3.

1. Give the definition for "sequential compactness" of a metric space.
2. Give the definition for "total boundedness" of a metric space.
3. Show that a sequentially compact metric space is totally bounded.

Problem 4.

1. Give the definition for a "path-connected set" in a metric space.
2. Show: Let (X, d) be a metric space. Let $\{A_\lambda \subseteq X \mid \lambda \in \Lambda\}$ be a collection of path-connected sets, such that $A_\lambda \cap A_\mu \neq \emptyset$ for all $\lambda,\ \mu \in \Lambda$. Show that $\bigcup\limits_{\lambda \in \Lambda} A_\lambda$ is path-connected. Be precise!

Problem 5.

1. Give the definition for "compactness" of a metric space.
2. Let (X, d) be a metric space, and let $A \subseteq X$. Show from first principles: If A is compact, then A is closed in X.