Homework 1 **Introduction to Analysis** **Spring 2001**

The problems are due on February 14.

Problem 1. A sequence (a_n) is called *proper*, if $a_n \neq a_m$ for all $n \neq m$. Show that a proper bounded sequence converges, if $\{a_n \mid n \in \mathbb{N}\}$ has **exactly one** accumulation point.

Problem 2. Let $\epsilon > 0$, and let (x_n) be a convergent sequence of real numbers. Show that there is a **Cauchy** sequence (r_n) of rational numbers satisfying $|x_n - r_n| \leq \epsilon$ for all $n \in \mathbb{N}$.

Problem 3. Let (a_n) be a sequence of real numbers. Set

$$b_n = \frac{1}{n}\left(a_1 + a_2 + a_3 + \cdots + a_n\right).$$

(a) Show: If (a_n) converges to $a \in \mathbb{R}$, then (b_n) converges to a.

(b) Show that the converse fails.