Homework 1 **Analysis** **Spring 2001**

The problems are due on Wednesday, February 14.

Problem 1. Show that $\mathbb{R}^n$ endowed with

$$\|(x_1, x_2, \ldots, x_n)\| := \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

is a normed vector space!

Problem 2. Consider C, the space of continuous real-valued functions on the unit interval $[0,1]$, endowed with the sup-norm $\|f\| = \sup_{x \in [0,1]} |f(x)|$. Let

$$A = \{f \in C \mid f(1/2) = 0\}$$

Is A open, closed? Find the interior, the closure and the boundary of A.

Problem 3. Recall the Bolzano-Weierstrass Theorem: *Every bounded infinite subset of real numbers has an accumulation point.*

Give an example to show that this result does not generalize to arbitrary metric spaces. (Remark: A subset A of a metric space (X, d) is called *bounded*, if there is an $x \in X$ and $R > 0$ such that $A \subseteq D(x, R)$.)