The problems are due on Wednesday, February 14.

**Problem 1.** Show that $\mathbb{R}^n$ endowed with

$$
\|(x_1, x_2, \ldots, x_n)\| := \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}
$$

is a normed vector space!

**Problem 2.** Consider $C$, the space of continuous real-valued functions on the unit interval $[0, 1]$, endowed with the sup-norm $\|f\| = \sup_{x \in [0,1]} |f(x)|$. Let

$$
A = \{ f \in C \mid f(1/2) = 0 \}
$$

Is $A$ open, closed? Find the interior, the closure and the boundary of $A$.

**Problem 3.** Recall the Bolzano-Weierstrass Theorem: *Every bounded infinite subset of real numbers has an accumulation point.*

Give an example to show that this result does not generalize to arbitrary metric spaces. (Remark: A subset $A$ of a metric space $(X, d)$ is called **bounded**, if there is an $x \in X$ and $R > 0$ such that $A \subseteq D(x, R)$.)