Homework 2 **Analysis** **Spring 2001**

The problems are due on Wednesday, February 28.

Problem 1. Let (x_n) and (y_n) be two Cauchy sequences in a metric space (X, d). Show that the sequence $(d(x_n, y_n))$ converges.

Problem 2. Consider the following two binary operations on $(0, 1]$:

$$d(x, y) = |x - y| \text{ for all } x, y \in (0, 1],$$

$$d^*(x, y) = \left|\frac{1}{x} - \frac{1}{y}\right| \text{ for all } x, y \in (0, 1].$$

1. Clearly d is a metric on $(0, 1]$. Show that d^* is also a metric on $(0, 1]$.
2. Show that both metrics define the same open sets.
3. Show that d^* is complete on $(0, 1]$, while d is *not* complete on $(0, 1]$.

Problem 3. A metric space (X, d) is called *separable* if it contains a countable dense subset. (The real numbers are separable, since the rational numbers form a countable dense subset.)

Show the following: If (X, d) is a separable metric space, and if $Y \subseteq X$, then (Y, d) is separable.