The problems are due on Wednesday, April 4.

Problem 1. Let \( (X, d) \) be a compact metric space, and \( f : X \rightarrow X \) be a map satisfying

\[
d(f(x), f(y)) = d(x, y) \quad \text{for all } x, y \in X.
\]

Denote by

\[
f^n := \underbrace{f \circ f \ldots \circ f}_{\text{n times}}.
\]

(a) Show that \( f \) is 1-1.
(b) Show that \( f \) is onto. Proceed as follows: If \( y \notin f(X) \), consider the sequence \( (f^n(y)) \). Show that this sequence contains no converging subsequence.

Problem 2. (a) Show that if \( K \) is a non-empty compact subset of a metric space \( (X, d) \), then for any \( x \in X \) there is a \( k \in K \), so that

\[
d(k, x) = \inf \{d(x, y) \mid y \in K\}.
\]

(b) Show that the conclusion in (a) fails if one replaces “compact” by “closed”.

Problem 3. Let \( D \) be a countable subset of \( \mathbb{R}^2 \). Show that the complement of \( D \), the set \( \mathbb{R}^2 \setminus D \), is pathwise connected!