Homework 4 **Analysis** **Spring 2001**

The problems are due on Wednesday, April 4.

Problem 1. Let (X, d) be a compact metric space, and $f : X \to X$ be a map satisfying

$$d(f(x), f(y)) = d(x, y) \text{ for all } x, y \in X.$$

Denote by

$$f^n := \underbrace{f \circ f \ldots \circ f}_{n \text{ times}}.$$

(a) Show that f is 1-1.
(b) Show that f is onto. Proceed as follows: If $y \notin f(X)$, consider the sequence $(f^n(y))$. Show that this sequence contains no converging subsequence.

Problem 2. (a) Show that if K is a non-empty compact subset of a metric space (X, d), then for any $x \in X$ there is a $k \in K$, so that

$$d(k, x) = \inf\{d(x, y) \mid y \in K\}.$$

(b) Show that the conclusion in **(a)** fails if one replaces "compact" by "closed".

Problem 3. Let D be a **countable** subset of $\mathbb{R}^2$. Show that the complement of D, the set $\mathbb{R}^2 \setminus D$, is pathwise connected!