The problems are due on Wednesday, April 18.

**Problem 1.** A set $A$ in a metric space $(X, d)$ is called an $F_\sigma$-set, if $A$ is the countable union of closed sets. Example: $\mathbb{Q}$ is an $F_\sigma$-set in $\mathbb{R}$.

Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function, and let $A \subseteq \mathbb{R}$ be an $F_\sigma$-set. Show that $f(A)$ is also an $F_\sigma$-set.

Does this result easily extend to arbitrary metric spaces? Explain!

**Problem 2.** For a real number $x$, define $x^+ = \max\{x, 0\}$ and $x^- = \max\{-x, 0\}$.

Let $(a_n)$ be a sequence of real numbers, such that $\sum a_n$ converges, while $\sum |a_n|$ diverges.

1. Show that the partial sums of $\sum a_n^+$ and the partial sums of $\sum a_n^-$ are unbounded increasing sequences. Note that $\lim_{n \to \infty} a_n^+ = 0$ and $\lim_{n \to \infty} a_n^- = 0$.

2. Let $L \in \mathbb{R}$. Using the results above, show that there is a bijection $\rho : \mathbb{N} \to \mathbb{N}$ such that

$$\sum_{n=1}^{\infty} a_{\rho(n)} = L.$$ 

**Problem 3.** Show: If $f : X \to Y$ is uniformly continuous, and $(x_n)$ is a Cauchy sequence in $X$, then $(f(x_n))$ is a Cauchy sequence in $Y$.

Is this result also true, if one only assumes that $f$ is continuous?