Homework 1 **Analysis** **Fall 2001**

The problems are due on Thursday, September 20.

For all students:

Problem 1. Let A be a subset of a metric space (X, d). Show that the following two definitions are equivalent:

[**1**] The interior of A is the set of all interior points of A.
[**2**] The interior of A is the union of all open sets contained in A.

Problem 2. Let Y be a subset of the metric space (X, d).

A. Show that (Y, d) is also a metric space.
B. Show that a set A is open in Y if and only if there is an open set O in X such that $A = O \cap Y$.

Problem 3. Recall that C, the space of real-valued continuous functions on $[0, 1]$, endowed with the sup-norm $\|f\| = \sup_{x \in [0,1]} |f(x)|$, is a normed vector space. Let

$$A = \{f \in C \mid f(1/2) \neq 0\}$$

Is A open, closed? Find the interior, the closure and the boundary of A.

For graduate students:

Problem 1G. Recall the Bolzano-Weierstrass Theorem: *Every bounded infinite subset of real numbers has an accumulation point.*

Does this result hold in arbitrary metric spaces? Give a proof, or find a counterexample. (Remark: A subset A of a metric space (X, d) is called *bounded*, if there is an $x \in X$ and $R > 0$ such that $A \subseteq D(x, R)$)

Problem 2G. A. Show that $A \subseteq \mathbb{R}$ is open iff A is the countable union of disjoint open intervals.
B. Is every closed subset of $\mathbb{R}$ the countable intersection of closed intervals?
C. Is a subset A in an arbitrary metric space (X, d) open iff A is the countable union of open balls?