1 Writing Down an Equation for a Line

Slope-Intercept-Form. Given the slope m and the y-intercept b of a line, an equation for the line can be written as

$$y = mx + b.$$

Example. Let a line have slope $m = -2$ and y-intercept $b = 3$. Then the line can be written as $y = -2x + 3$.

Point-Slope-Form. Given the slope m and a point (x_0, y_0) on the line, an equation for the line can be written as

$$y - y_0 = m(x - x_0).$$

Two-Point-Form. If you know two points (x_0, y_0) and (x_1, y_1) on a line, its equation can be written as

$$y - y_0 = \frac{y_1 - y_0}{x_1 - x_0} \cdot (x - x_0).$$

Note that the expression $\frac{y_1 - y_0}{x_1 - x_0}$ is nothing else but the slope of the line.

Example. Suppose a line passes through the points $(x_0, y_0) = (1, 0)$ and $(x_1, y_1) = (-2, 4)$. Then we can write an equation of the line as

$$y - 0 = \frac{4 - 0}{(-2) - 1} \cdot (x - 1),$$

or simplified as:

$$y = -\frac{4}{3}(x - 1).$$

2 Solving Linear Equations

- Strategy: Move all "x" items to one side of the equation, the rest to the other side of the equation.
- You are allowed to do the following:

1. Add (or subtract) the same number on both sides.
2. Multiply (or divide) by the same number on both sides. Do **not** multiply or divide both sides by 0!

Example 1.

$$2x - 7 = 4$$

Add 7 on both sides.

$$\begin{aligned} 2x - 7 &= 4 && \Big| \quad +7 \\ 2x - 7 + 7 &= 4 + 7 \\ 2x &= 11. \end{aligned}$$

Divide both sides by 2.

$$\begin{aligned} 2x &= 11 && \Big| \quad \div 2 \\ x &= \frac{11}{2} \end{aligned}$$

Check that $x = \dfrac{11}{2}$ is a solution of the original equation!

Example 2.

$$\frac{2}{3}x - \frac{4}{5} = \frac{1}{5}x + \frac{1}{3}$$

Add $\dfrac{4}{5}$ on both sides.

$$\begin{aligned} \frac{2}{3}x - \frac{4}{5} &= \frac{1}{5}x + \frac{1}{3} && \Big| \quad +\frac{4}{5} \\ \frac{2}{3}x &= \frac{1}{5}x + \frac{1}{3} + \frac{4}{5} \end{aligned}$$

Subtract $\dfrac{1}{5}x$ on both sides.

$$\begin{aligned} \frac{2}{3}x &= \frac{1}{5}x + \frac{1}{3} + \frac{4}{5} && \Big| \quad -\frac{1}{5}x \\ \frac{2}{3}x - \frac{1}{5}x &= \frac{1}{3} + \frac{4}{5} \end{aligned}$$

Simplify both sides.

$$\frac{2}{3}x - \frac{1}{5}x = \frac{1}{3} + \frac{4}{5}$$
$$\frac{7}{15}x = \frac{17}{15}$$

Multiply both sides by $\dfrac{15}{7}$ and simplify.

$$\begin{aligned} \frac{7}{15}x &= \frac{17}{15} & \Big| \cdot\frac{15}{7} \\ \left(\frac{7}{15}\cdot\frac{15}{7}\right)x &= \frac{17}{15}\cdot\frac{15}{7} \\ x &= \frac{17}{7} \end{aligned}$$

Check that $x = \dfrac{17}{7}$ is a solution of the original equation!

3 Linear Regression: Finding the Best Line Fitting the Data – An Example

1. Start with the data table; in the example given here, the number of data points is $n = 3$:

x	y
1	2
2	1
4	-2

2. Compute the average ave(x) of the x-coordinates, and the average ave(y) of the y-coordinates:

$$\begin{aligned} \text{ave}(x) &= (x_1 + x_2 + \cdots + x_{n-1} + x_n)/n = (1+2+4)/3 = 2.33 \\ \text{ave}(y) &= (y_1 + y_2 + \cdots + y_{n-1} + y_n)/n = (2+1+(-2))/3 = 0.33 \end{aligned}$$

3. Compute the deviations from the averages:

x	y	x-ave(x)	y-ave(y)
1	2	-1.33	1.67
2	1	-0.33	0.67
4	-2	1.67	-2.33

4. Compute S_{xx} and S_{xy}:

$$\begin{aligned} S_{xx} &= (x_1 - \text{ave}(x))^2 + (x_2 - \text{ave}(x))^2 + \cdots + (x_{n-1} - \text{ave}(x))^2 + (x_n - \text{ave}(x))^2 \\ &= 4.67 \\ S_{xy} &= (x_1 - \text{ave}(x))(y_1 - \text{ave}(y)) + (x_2 - \text{ave}(x))(y_2 - \text{ave}(y)) + \cdots \\ &\quad \cdots + (x_n - \text{ave}(x))(y_n - \text{ave}(y)) \\ &= -6.33 \end{aligned}$$

x	y	x-ave(x)	y-ave(y)	(x-ave(x))^2	(x-ave(x))·(y-ave(y))
1	2	-1.33	1.67	1.77	-2.22
2	1	-0.33	0.67	0.11	-0.22
4	-2	1.67	-2.33	2.79	-3.89
				S_{xx} =4.67	S_{xy}=-6.33

5. The slope m of the regression line is

$$m = \frac{S_{xy}}{S_{xx}} = \frac{-6.33}{4.67} = -1.36$$

The y-intercept of the regression line is given by

$$b = \text{ave}(y) - m \cdot \text{ave}(x) = 0.33 - (-1.36)(2.33) = 3.50$$

Thus the regression line has the equation

$$y = mx + b = -1.36x + 3.50.$$