Homework 1 **Introduction to Analysis** **Spring 2003**

The problems are due on February 12.

Problem 1. Let S be a non-empty set, which is bounded from above. Show that there is a sequence (s_n) of elements in S that converges to the supremum of S.

Problem 2. A sequence (a_n) is called decreasing if $a_n \leq a_m$ whenever $n \geq m$. Show that every **bounded** decreasing sequence converges.

Problem 3. Let r be a real number. Show that there is a sequence (q_n) of rational numbers such that (q_n) converges to r.

Problem 4. Let (a_n) be a sequence of real numbers. Set

$$b_n = \frac{1}{n}\left(a_1 + a_2 + a_3 + \cdots + a_n\right).$$

(a) Show: If (a_n) converges to $a \in \mathbb{R}$, then (b_n) converges to a.

(b) Show that the converse fails.