Homework 2 **Introduction to Analysis** **Spring 2003**

The problems are due on March 5.

The problems below use the following definitions: Let (a_n) be a bounded sequence of real numbers. We define the *limes inferior* and *limes superior* of the sequence as

$$\liminf_{n\to\infty} a_n := \lim_{k\to\infty} \left(\inf\{a_n \mid n \geq k\}\right),$$

and

$$\limsup_{n\to\infty} a_n := \lim_{k\to\infty} \left(\sup\{a_n \mid n \geq k\}\right).$$

Problem 1. Explain why the numbers $\liminf_{n\to\infty} a_n$ and $\limsup_{n\to\infty} a_n$ are well-defined for every bounded sequence (a_n).

Problem 2. Show that a bounded sequence (a_n) converges if and only if

$$\liminf_{n\to\infty} a_n = \limsup_{n\to\infty} a_n.$$

Problem 3. Let (a_n) be a bounded sequence of real numbers. Show that (a_n) has a subsequence that converges to $\limsup_{n\to\infty} a_n$.

Problem 4. Let (a_n) be a bounded sequence of real numbers, and let (a_{n_k}) be one of its converging subsequences. Show that

$$\liminf_{n\to\infty} a_n \leq \lim_{k\to\infty} a_{n_k} \leq \limsup_{n\to\infty} a_n.$$