

Introduction:

Why Cognitive Science Matters to Mathematics

MATHEMATICS AS WE KNOW IT HAS BEEN CREATED and used by human beings: mathematicians, physicists, computer scientists, and economists—all members of the species *Homo sapiens*. This may be an obvious fact, but it has an important consequence. Mathematics as we know it is limited and structured by the human brain and human mental capacities. The only mathematics we know or can know is a brain-and-mind-based mathematics.

As cognitive science and neuroscience have learned more about the human brain and mind, it has become clear that the brain is not a general-purpose device. The brain and body co-evolved so that the brain could make the body function optimally. Most of the brain is devoted to vision, motion, spatial understanding, interpersonal interaction, coordination, emotions, language, and everyday reasoning. Human concepts and human language are not random or arbitrary; they are highly structured and limited, because of the limits and structure of the brain, the body, and the world.

This observation immediately raises two questions:

1. Exactly what mechanisms of the human brain and mind allow human beings to formulate mathematical ideas and reason mathematically?
2. Is brain-and-mind-based mathematics all that mathematics is? Or is there, as Platonists have suggested, a disembodied mathematics transcending all bodies and minds and structuring the universe—this universe and every possible universe?

Question 1 asks where mathematical ideas come from and how mathematical ideas are to be analyzed from a cognitive perspective. Question 1 is a scientific question, a question to be answered by cognitive science, the interdisciplinary science of the mind. As an empirical question about the human mind and brain, it cannot be studied purely within mathematics. And as a question for empirical science, it cannot be answered by an a priori philosophy or by mathematics itself. It requires an understanding of human cognitive processes and the human brain. Cognitive science matters to mathematics because only cognitive science can answer this question.

Question 1 is what this book is mostly about. We will be asking how normal human cognitive mechanisms are employed in the creation and understanding of mathematical ideas. Accordingly, we will be developing techniques of mathematical idea analysis.

But it is Question 2 that is at the heart of the philosophy of mathematics. But it is the question that most people want answered. Our answer is straightforward:

- Theorems that human beings prove are within a human mathematical conceptual system.
- All the mathematical knowledge that we have or can have is knowledge within human mathematics.
- There is no way to know whether theorems proved by human mathematicians have any objective truth, external to human beings or any other beings.

The basic form of the argument is this:

1. The question of the existence of a Platonic mathematics cannot be addressed scientifically. At best, it can only be a matter of faith, much like faith in a God. That is, Platonic mathematics, like God, cannot in itself be perceived or comprehended via the human body, brain, and mind. Science alone can neither prove nor disprove the existence of a Platonic mathematics, just as it cannot prove or disprove the existence of a God.
2. As with the conceptualization of God, all that is possible for human beings is an understanding of mathematics in terms of what the human brain and mind afford. The only conceptualization that we can have of mathematics is a human conceptualization. Therefore, mathematics as we know it and teach it can only be humanly created and humanly conceptualized mathematics.
3. What human mathematics is, is an empirical scientific question, not a mathematical or a priori philosophical question.
4. Therefore, it is only through cognitive science—the interdisciplinary study of mind, brain, and their relation—that we can answer the question: What is the nature of the only mathematics that human beings know or can know?
5. Therefore, if you view the nature of mathematics as a scientific question, then mathematics is mathematics as conceptualized by human beings using the brain's cognitive mechanisms.
6. However, you may view the nature of mathematics itself not as a scientific question but as a philosophical or religious one. The burden of scientific proof is on those who claim that an external Platonic mathematics does exist, and that theorems proved in human mathematics are objectively true, external to the existence of any beings or any conceptual systems, human or otherwise. At present there is no known way to carry out such a scientific proof in principle.

From: George Lakoff, Rafael E. Núñez, *Where Mathematics Comes From*, Basic Books 2000.