Test 2

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(1) Let $X \in \mathcal{P}(B \setminus A) \setminus \{\emptyset\}$. Thus $X \subseteq B \setminus A$ and $X \neq \emptyset$. Let $y \in X$. Then $y \in B$ and $y \notin A$; thus $X \nsubseteq A$. Since $X \subseteq B$, it follows that $X \in \mathcal{P}(B) \setminus \mathcal{P}(A)$.

(2) $\bigcup M_n = \mathbb{Z}$

"$\subseteq$" obvious since $M_n \subseteq \mathbb{Z}$ $\forall n \in \mathbb{N}$

"$\supseteq$" Note that if $z = 0$, $z \in M_1$; if $z \neq 0$, then $z \in M_{|z|}$

$\bigcap M_n = \{0\}$

"$\subseteq$" If $z \neq 0$, then $z \notin M_{2|z|}$, thus $z \notin \bigcap M_n$

"$\supseteq$" obvious since $0 \in M_n$ $\forall n \in \mathbb{N}$.

(4) Induction step: Let $n \in \mathbb{N}$ be given. Suppose all $k \leq n$ can be written as products of primes. If $n+1$ is prime, we are done; so let's assume $n+1$ is not prime. Then $n+1 = a \cdot b$ for some $a, b \in \mathbb{N}$ with $1 < a \leq n$ and $1 < b \leq n$. By our induction hypothesis we can write both a and b as products of primes:

$a = p_1 \cdot p_2 \cdot \cdots p_j$; $b = q_1 \cdot q_2 \cdot \cdots q_k$

Therefore $n+1 = p_1 \cdot p_2 \cdot \cdots p_j \cdot q_1 \cdot \cdots q_k$ is a product of primes.

(5) (a) reflexivity ✓ $a \sim a$ for all $a \in \mathbb{Z}$
since $a = \underline{1} \cdot a$
Symmetry X $4 \sim 2$ but $2 \nsim 4$
anti-symm X $4 \sim -4$ and $-4 \sim 4$.
transitivity ✓ If $a = kb$ and $b = lc$,
then $a = (k \cdot l) c$ (with $k \cdot l \in \mathbb{Z}$)

(b) reflexivity X $2 \nsim 2$
Symmetry X $4 \sim 2$ but $2 \nsim 4$
anti-symm. ✓ $a \sim_p b$ and $b \sim_q a$
implies $a = (pq) a$. Since $pq \neq 1$,
this implies $a = 0$ and therefore
$b = 0$.
transitivity X the product of 2
primes is not a prime
number.