The assignment is due at the beginning of class on March 20, 2006.

Problem 1 (10 points) Let A be a set, and let $\mathcal{K}$ be a collection of sets. Show that

$$A \cup \Big(\bigcap_{B \in \mathcal{K}} B\Big) = \bigcap_{B \in \mathcal{K}} (A \cup B).$$

Problem 2 (10 points) Prove for all $n \in \mathbb{N}$:

$$1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Problem 3 (10 points) Prove for all natural numbers $n \geq 5$: $2^n > n^2$.

Problem 4 (10 points) Let $n \in \mathbb{N}$. Conjecture a formula for the expression

$$a_n = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)}$$

and prove your conjecture by induction.

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