The assignment is due at the beginning of class on April 3, 2006.

**Problem 1 (10 points)** Let \( R \) and \( S \) be two relations on \( \mathbb{R} \): \( R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y < x^2\} \) and \( S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = 2x - 1\} \). Find \( S \circ R \) and \( R \circ S \).

**Problem 2 (10 points)** Let \( R \) be a relation from \( A \) to \( B \). For an element \( b \in B \) define the set \( R_b := \{a \in A \mid (a, b) \in R\} \). Show \( \bigcup_{b \in B} R_b = \text{Dom } R \).

**Problem 3 (10 points)** Define a relation \( R \) on \( \mathbb{R} \) as follows: \( a \, R \, b \) if \( a - b \) is irrational. Prove or disprove: \( R \) is (a) reflexive, (b) symmetric, (c) transitive.

**Problem 4 (10 points)** Let \( K \) be the following subset of \( \mathbb{Z} \times \mathbb{Z} \):

\[ K = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid y \neq 0\}, \]

and define a relation \( \sim \) on \( K \) by setting \( (a, b) \sim (c, d) \) if \( ad = bc \). Show that \( \sim \) is an equivalence relation on \( K \).