Math 3325 **Homework 6** **Spring 2006**

The assignment is due at the beginning of class on April 3, 2006.

Problem 1 (10 points) Let R and S be two relations on $\mathbb{R}$: $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y < x^2\}$ and $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = 2x - 1\}$. Find $S \circ R$ and $R \circ S$.

Problem 2 (10 points) Let R be a relation from A to B. For an element $b \in B$ define the set $R_b := \{a \in A \mid (a, b) \in R\}$. Show

$$\bigcup_{b \in B} R_b = \operatorname{Dom} R.$$

Problem 3 (10 points) Define a relation R on $\mathbb{R}$ as follows: $a\,R\,b$ if $a - b$ is irrational. Prove or disprove: R is (a) reflexive, (b) symmetric, (c) transitive.

Problem 4 (10 points) Let K be the following subset of $\mathbb{Z} \times \mathbb{Z}$:

$$K = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid y \neq 0\},$$

and define a relation $\sim$ on K by setting $(a, b) \sim (c, d)$ if $ad = bc$. Show that $\sim$ is an equivalence relation on K.