Math 3325  Homework 7  Spring 2006

The assignment is due at the beginning of class on April 10, 2006.

Problem 1 (10 points) Consider the following relation on the set $A = \{1, 2, 3, 4, 5, 6\}$:

$$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,2), (1,4), (2,1), (2,4), (4,1), (4,2), (3,6), (6,3)\}.$$  

Find the partition generated by $R$.

For the next two problems we adopt the following notation: Let $A$ be a non-empty set, and let $C \subseteq \mathcal{P}(A)$ with $C \neq \emptyset$. We say that a relation $R$ on $A$ is the relation generated by $C$, if

$$aRb \iff \exists D \in C \text{ such that } a, b \in D.$$  

Note that the relation generated by a collection is always symmetric.

Problem 2 (10 points) Prove or disprove: If $D \neq \emptyset$ for all $D \in C$, then the relation $R$ generated by $C$ is reflexive.

Problem 3 (10 points) Prove or disprove: If $C$ is a pairwise disjoint collection, then the relation $R$ generated by $C$ is transitive.

Problem 4 (10 points) Show that the relation $R$ on $\mathbb{N}$ defined by

$$aRb \iff \text{there is a non-negative integer } k \text{ such that } b = 2^k a$$

is a partial ordering.